# Degrees and radians

Introduction

Angles can be measured in units of either degrees or radians. This leaflet explains these units and shows how it is possible to convert between them.

1. Degrees and radians

Angles can be measured in units of either degrees or radians. The symbol for degree is $^\circ$. Usually no symbol is used to denote radians.

A complete revolution is defined as 360° or 2π radians. π stands for the number $3.14159\ldots$ and you can work with this if you prefer. However in many calculations you will find that you need to work directly with multiples of π.

half a revolution $=180^\circ=\pi$ radians

complete revolution $= 360^\circ = 2\pi$ radians
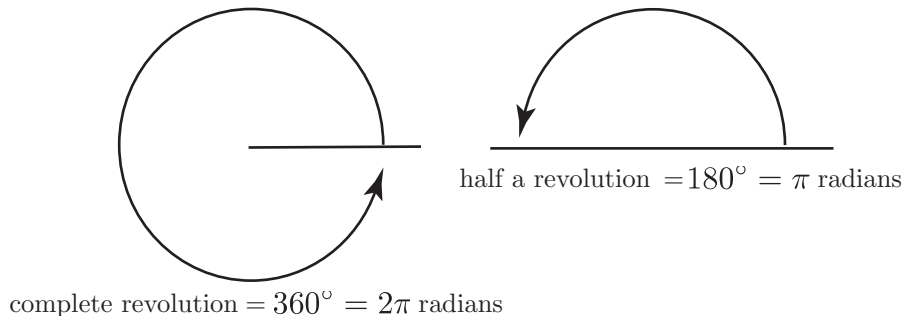
It is easy to use the fact that $360^\circ = 2\pi$ radians to convert between the two measures. We have

$$
\begin{aligned}
360^\circ &= 2\pi \text{ radians} \\
1^\circ &= \frac{2\pi}{360} = \frac{\pi}{180} \text{ radians} \\
1 \text{ radian} &= \frac{180}{\pi} \text{ degrees} \approx 57.3^\circ
\end{aligned}
$$

Example

a) Convert 65° to radians.   b) Convert 1.75 radians to degrees.
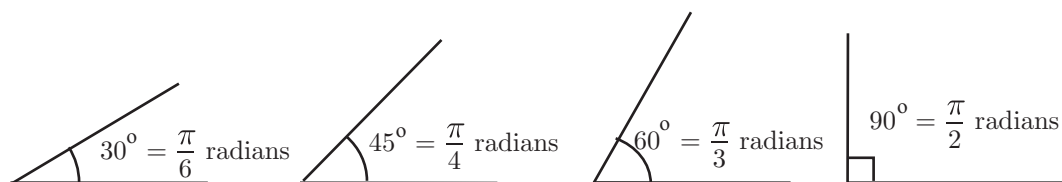
Solution

a)

$$1° = \frac{\pi}{180} \text{ radians}$$
$$65° = 65 \times \frac{\pi}{180}$$
$$= 1.134 \text{ radians}$$

b)

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$
$$1.75 \text{ radians} = 1.75 \times \frac{180}{\pi}$$
$$= 100.268°$$

Note the following commonly met angles:

$$30° = \tfrac{\pi}{6} \text{ radians} \qquad 45° = \tfrac{\pi}{4} \text{ radians} \qquad 60° = \tfrac{\pi}{3} \text{ radians}$$
$$90° = \tfrac{\pi}{2} \text{ radians} \quad 135° = \tfrac{3\pi}{4} \text{ radians} \quad 180° = \pi \text{ radians}$$

$30° = \frac{\pi}{6}$ radians $45° = \frac{\pi}{4}$ radians $60° = \frac{\pi}{3}$ radians $90° = \frac{\pi}{2}$ radians

Your calculator should be able to work with angles measured in both radians and degrees. Usually the MODE button allows you to select the appropriate measure. When calculations involve calculus you should always work with radians and not degrees.

Exercises

1. Convert each of the following angles given in degrees, to radians. Give your answers correct to 2 decimal places.

a) 32°,   b) 95°,   c) 217°.

2. Convert each of the following angles given in radians, to degrees. Give your answers correct to 2 decimal places.

a) 3 radians,   b) 2.4 radians,   c) 1 radian.

3. Convert each of the following angles given in radians, to degrees. Do not use a calculator.

a) $\frac{\pi}{15}$,   b) $\frac{\pi}{5}$.

4. Convert the following angles given in degrees, to radians. Do not use a calculator and give your answers as multiples of π.

a) 90°,   b) 72°,   c) −45°.

Answers

1. a) 0.56 radians, b) 1.66 radians, c) 3.79 radians.   2. a) 171.89°,   b) 137.51°,   c) 57.30°.
3. a) 12°,   b) 36°.   4. a) $\frac{\pi}{2}$ radians,   b) $\frac{2\pi}{5}$ radians,   c) $-\frac{\pi}{4}$ radians.