3   Session Three Functions, Graphs, Differentiation

3.1 What is a Function?

Questions:

1. Given $f(x) = x^2 + 2x + 3$, calculate $f(5)$.

2. Find the range of values for $g(x)$ if $g(x) = \dfrac{1}{x+2}$ and $3 \leq x \leq 8$.

3. Given $h(x) = x^2 + 1$, find the range of values for x such that $5 \leq h(x) \leq 10$.

Answers: 1.   38 ,   2. $0.1 \leq f(x) \leq 0.2$,   3. $-3 \leq x \leq -2$ or $2 \leq x \leq 3$

3.5 Basic differentiation

Questions: Find the derivatives of the following:

1. $y = 5x^2$ $A: \quad \dfrac{dy}{dx} = 10x$

2. $f(x) = 2x^3 + 4x^{-2}$ $A: \quad f'(x) = 6x^2 - 8x^{-3}$

3. $z = 7y^3 + 6y^2 + 5y + 4$ $A: \quad \dfrac{dz}{dx} = 21y^2 + 12y + 5$

4. $p(x) = \pi$ $A: \quad p'(x) = 0$

5. $g(t) = t^{0.5} - t^{-0.5}$ $A: \quad g'(t) = \frac{1}{2} t^{-0.5} + \frac{1}{2} t^{-1.5}$

6. $s = \dfrac{4}{x^3} - \dfrac{2}{3}\sqrt{x}$ $A: \quad \dfrac{ds}{dx} = -\dfrac{12}{x^4} - \dfrac{1}{3\sqrt{x}}$

7. $y = 2\sin x + 5e^x$ $A: \quad \dfrac{dy}{dx} = 2\cos x + 5e^x$

8. $z = 4\tan t - \dfrac{2}{5t^3}$ $A: \quad \dfrac{dz}{dt} = \dfrac{4}{\cos^2 t} + \dfrac{6}{5t^4}$

9. $f(t) = a\,\cos t + b\,\sin t$ $A: \quad f'(t) = -a\,\sin t + b\,\cos t$

10. $g(x) = 4\ln x + \ln e^{3x^4}$ $A: \quad g'(x) = \dfrac{4}{x} + 12x^3$

11. $h(x) = \sqrt[5]{x^3} - e^{6x}$ $A: \quad h'(x) = \dfrac{3}{5}\dfrac{1}{\sqrt[5]{x^2}} - 6e^{6x}$

12. $y = \ln 4t - \ln\dfrac{t}{3}$ $A: \quad \dfrac{dy}{dt} = \dfrac{1}{t} - \dfrac{1}{t} = 0$

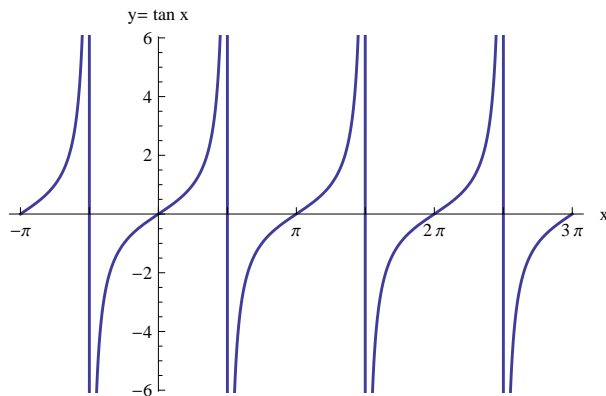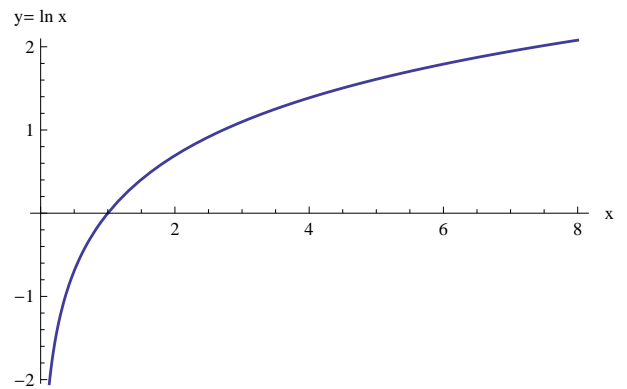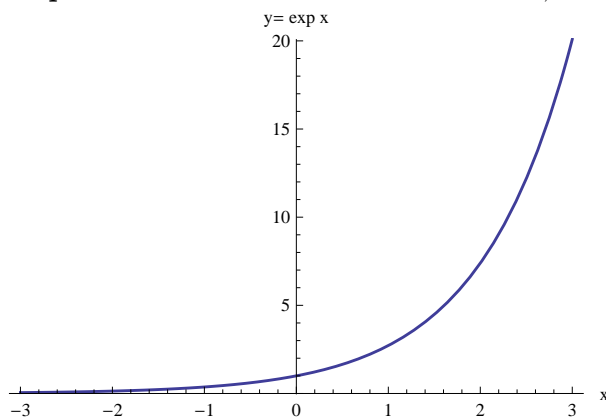## 3.2   Trigonometric, Exponential and Logarithm Functions

Sine Function

y= sin x

Cosine Function

y= cos x

Tangent Function

y= tan x
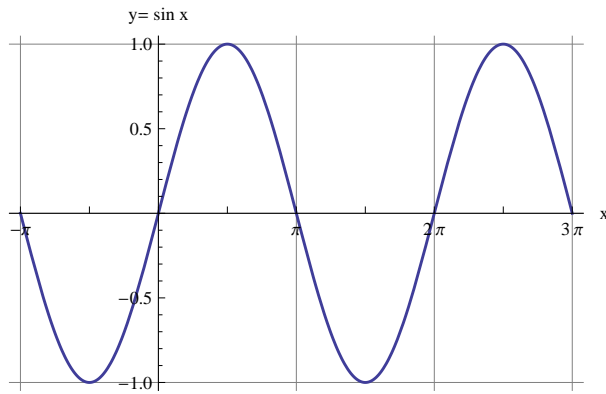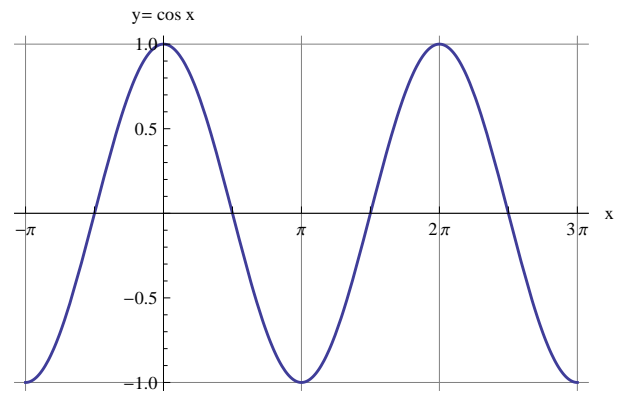
Exponential Function $e = 2.71828\ldots$, $e^3 \approx 20$ **Natural Logarithm Function** $\ln 8 \approx 2$

y= exp x

y= ln x

$e = 2.7\,1828\,1828\,459045\ldots$

$\pi = 3.141592\,6535\,8979\,3238\,4626\ldots$