A Calculus Refresher

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1. Derivatives of basic functions

1. Differentiate each of the following with respect to x.

(a) x (b) x^6 (c) 6 (d) $\sqrt{x}$ (e) x^{-1} (f) $x^{1/7}$

(g) $\dfrac{1}{x^3}$ (h) x^{79} (i) $x^{1.3}$ (j) $\dfrac{1}{\sqrt[3]{x}}$ (k) $x^{-5/3}$ (l) $\dfrac{1}{x^{0.71}}$

2. Differentiate each of the following with respect to θ.

(a) $\cos\theta$ (b) $\cos 4\theta$ (c) $\sin\theta$ (d) $\sin\dfrac{2\theta}{3}$ (e) $\tan\theta$ (f) $\tan\pi\theta$

(g) $\sin(-8\theta)$ (h) $\tan\dfrac{\theta}{4}$ (i) $\cos 3\pi\theta$ (j) $\cos\left(-\dfrac{5\theta}{2}\right)$ (k) $\sin 0.7\theta$

3. Find the following derivatives.

(a) $\dfrac{d}{dx}(e^x)$ (b) $\dfrac{d}{dy}(e^{2y})$ (c) $\dfrac{d}{dt}(e^{-7t})$ (d) $\dfrac{d}{dx}(e^{-x/3})$

(e) $\dfrac{d}{dz}(e^{2z/\pi})$ (f) $\dfrac{d}{dx}(e^{-1.4x})$ (g) $\dfrac{d}{dx}(3^x)$

4. Find the following derivatives.

(a) $\dfrac{d}{dx}(\ln x)$ (b) $\dfrac{d}{dz}(\ln 5z)$ (c) $\dfrac{d}{dx}\left(\ln\dfrac{2x}{3}\right)$

2. Linearity in differentiation

The **linearity rules** enable us to differentiate sums and differences of functions, and constant multiples of functions. Specifically

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x)), \qquad \frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x)).$$

1. Differentiate each of the following with respect to x.

(a) $3x + 2$ (b) $2x - x^2$ (c) $-\cos x - \sin x$ (d) $3x^{-3} + 4\sin 4x$

(e) $2e^x + e^{-2x}$ (f) $\dfrac{1}{x} - 4 - 3\ln x$ (g) $4x^5 - 3\tan 8x - 2e^{5x}$

2. Find the following derivatives.

(a) $\dfrac{d}{dt}\left(5t^{1/5} + \dfrac{t^8}{8}\right)$ (b) $\dfrac{d}{d\theta}\left(2\cos\dfrac{\theta}{4} - 3e^{-\theta/4}\right)$ (c) $\dfrac{d}{dx}\left(\dfrac{3e^{3x/5}}{5}\right)$

(d) $\dfrac{d}{dx}\left(\dfrac{2}{9}\tan\dfrac{3x}{2} - \dfrac{3}{4}\cos 8x\right)$ (e) $\dfrac{d}{dz}\left(\dfrac{1}{4}z^{4/3} - \dfrac{1}{3}e^{-4z/3}\right)$

In Questions 3-5 you don't need the product rule, quotient rule or chain rule to differentiate any of these if you do the algebra first!

3. Expand the powers or roots and hence find the following derivatives.

(a) $\dfrac{d}{dy}\left(\sqrt{2y}\right)$ (b) $\dfrac{d}{dx}\left((2x)^3 - \dfrac{1}{(2x)^3}\right)$ (c) $\dfrac{d}{dy}\left(\left(\dfrac{1}{2}e^y\right)^4\right)$ (d) $\dfrac{d}{dt}\left(\sqrt[3]{5e^{-2t}}\right)$

4. Simplify or expand each of the following expressions, and then differentiate with respect to x.

(a) $\dfrac{x - x^2}{x^3}$ (b) $x(\sqrt{x} - x^2)$ (c) $\left(2x - \dfrac{2}{x}\right)\left(\dfrac{3}{x^2} + x\right)$

(d) $(e^{2x} - 1)(3 - e^{3x})$ (e) $\dfrac{1 - e^{-2x}}{e^{-4x}}$

5. Use the laws of logarithms to find the following derivatives.

(a) $\dfrac{d}{dx}\left(\ln x^{9/2}\right)$ (b) $\dfrac{d}{dx}\left(\ln\left(\dfrac{1}{\sqrt{6x}}\right)\right)$ (c) $\dfrac{d}{dt}\left(\ln\left(\dfrac{t^3}{e^{3t}}\right)\right)$ (d) $\dfrac{d}{dt}\left(\ln\left(te^{-2t}\right)^{1/3}\right)$

3. Higher derivatives

1. Find the following second derivatives.

(a) $\dfrac{d^2}{dx^2}(x^5)$ (b) $\dfrac{d^2}{dx^2}(\cos 3x)$ (c) $\dfrac{d^2}{dz^2}(e^{2z} - e^{-2z})$

(d) $\dfrac{d^2}{dy^2}(8 - 13y)$ (e) $\dfrac{d^2}{dx^2}\left(\dfrac{1}{x} - 3x - 3x^3\right)$ (f) $\dfrac{d^2}{dt^2}(\ln 2t - \sqrt{6t})$

(g) $\dfrac{d^2}{dx^2}\left(x^{3/2} - \dfrac{1}{x^{3/2}}\right)$ (h) $\dfrac{d^2}{dx^2}(e^x + e^{-x} + \sin x + \cos x)$ (i) $\dfrac{d^2}{dt^2}\left(\dfrac{1}{2}\sin 2t - \dfrac{1}{4}\ln 4t\right)$