2 Session Two - Complex Numbers and Vectors

Workshop Questions

1. Determine the sums (a) $(3+\mathrm{i})+(3-2\,\mathrm{i})$ and (b) $(-5+2\,\mathrm{i})+(-3+4\,\mathrm{i})$.

2. Find the products (a) $(2+2\,\mathrm{i})\cdot(-1-\mathrm{i})$ and (b) $(2+2\,\mathrm{i})\cdot(2-2\,\mathrm{i})$.

3. Plot the following on an Argand diagram, and find the modulus and argument of each
$$\text{(a) } z=-2\ , \quad \text{(b) } z=2+2\,\mathrm{i}\ , \quad \text{(c) } z=-1-\mathrm{i}\ , \quad \text{(d) } z=6\,\mathrm{i}\ .$$

4. Find the real and imaginary parts of (a) $z=3\,e^{5\mathrm{i}}$ and (b) $z=2\,e^{\pi\mathrm{i}}$.

5. Express the complex number $(2+2\,\mathrm{i})$ in (a) polar and (b) exponential form.

6. Express the complex number $(3-4\,\mathrm{i})$ in (a) polar and (b) exponential form.

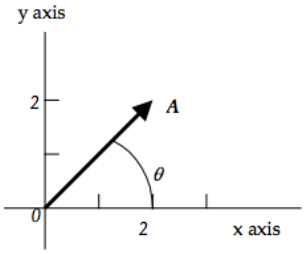
7. Plot on an Argand diagram the numbers $z_1=3$ and $z_2=2\,\mathrm{i}$. Determine their sum z_3 and plot this on the same diagram. Find the arguments of z_1, z_2, and their sum. Qualitatively, how would the argument of the sum change if the modulus of z_2 was reduced?

8. What is the complex conjugate of (a) $z=(4+3\,\mathrm{i})$ and (b) $z=(1-g)e^{g(2+2\,\mathrm{i})}$ (g is real)?

9. By multiplying together $z_1=a+\mathrm{i}\,b$ and $z_2=a-\mathrm{i}\,b$, show that one is the complex conjugate of the other. Do the same with $z_3=g^{2x+\mathrm{i}y}$ and $z_4=g^{2x-\mathrm{i}y}$ to show that one is the complex conjugate of the other (g is real).

10. Evaluate the quotients (a) $\dfrac{2+2\,\mathrm{i}}{3+4\,\mathrm{i}}$ and (b) $\dfrac{2-4\,\mathrm{i}}{1-\mathrm{i}}$.

11. (a) What is the Cartesian form of the vector **A** shown in the diagram below?
    (b) What is the direction this vector makes with the x-axis?
    (c) What is the magnitude of the vector?



12. Sketch the vector $\mathbf{B} = (3\,\hat{\mathbf{i}} - \hat{\mathbf{j}})$.

13. Sketch the vector $\mathbf{C} = (3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})$.

14. Sketch the vector $\mathbf{D} = (3\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}})$.

15. Evaluate (a) $\mathbf{B} + \mathbf{C}$ and (b) $\mathbf{A} + \mathbf{B}$.

16. Determine $3\,\mathbf{C}$.

17. Evaluate (a) $\mathbf{A} - \mathbf{B}$ and (b) $\mathbf{C} - \mathbf{B}$.

18. Find the magnitude and direction of $2\,\mathbf{B}$.

19. Evaluate (a) $\mathbf{B} \cdot \mathbf{C}$ (b) $\mathbf{C} \cdot \mathbf{B}$ (c) $\mathbf{A} \cdot \mathbf{B}$ (d) $\mathbf{C} \cdot \mathbf{C}$

20. Evaluate (a) $\mathbf{B} \times \mathbf{C}$ (b) $\mathbf{C} \times \mathbf{B}$ (c) $\mathbf{B} \times \mathbf{A}$ (d) $\mathbf{C} \times \mathbf{C}$

21. Evaluate (a) $\mathbf{B} \cdot \mathbf{D}$ (b) $\mathbf{B} \times \mathbf{D}$

22. Express Cartesian coordinates (a) $(3, 4)$ and (b) $(-2, 1)$ in (r, θ) polar coordinates.