Valuation Risk Revalued

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ABSTRACT

This paper shows the recent success of valuation risk (time-preference shocks in Epstein-Zin utility) in resolving asset pricing puzzles rests sensitively on an undesirable asymptote that occurs because the preference specification fails to satisfy a key restriction on the weights in the Epstein-Zin time-aggregator. In a Bansal-Yaron long-run risk model, our revised valuation risk specification that satisfies the restriction provides a superior empirical fit. The results also show that valuation risk no longer has a major role in matching the mean equity premium and risk-free rate but is crucial for matching the volatility and autocorrelation of the risk-free rate.

Keywords: Epstein-Zin Utility; Valuation Risk; Equity Premium Puzzle; Risk-Free Rate Puzzle
JEL Classifications: D81; G12

1 INTRODUCTION

In standard asset pricing models, uncertainty enters through the supply side of the economy, either through endowment shocks in a Lucas (1978) tree model or productivity shocks in a production economy model. Recently, several papers introduced demand side uncertainty or "valuation risk" as a potential explanation of key asset pricing puzzles (Albuquerque et al. (2016, 2015); Creal and Wu (2017); Maurer (2012); Nakata and Tanaka (2016); Schorfheide et al. (2018)). In macroeconomic parlance, valuation risk is usually referred to as a discount factor or time preference shock.[1]

The literature contends valuation risk is an important determinant of key asset pricing moments when it is embedded in Epstein and Zin (1991) recursive preferences. We show the success of valuation risk rests on an undesirable asymptote that permeates the determination of asset prices. The influence of the asymptote is easily identified in a stylized model. In that model, an intertemporal elasticity of substitution (IES) marginally above one predicts an arbitrarily large equity premium and an arbitrarily low risk-free rate, while an IES slightly below one predicts the opposite results. Moreover, the asymptote significantly affects equilibrium outcomes even when the IES is well above unity by qualitatively changing the relationship between the IES and the equity premium.

de Groot et al. (2018) show that with Epstein-Zin preferences, time-varying weights in a CES time-aggregator must sum to 1 to prevent an undesirable asymptote from determining equilibrium outcomes. The current specification used in the literature fails to impose this important restriction. de Groot et al. (2018) propose an alternative specification (henceforth, the "revised specification") that eliminates the asymptote and ensures that preferences are well-defined when the IES is one.[2]

This paper uses the revised specification to re-evaluate the role of valuation risk in explaining key asset pricing moments. While the change to the model will appear minor, it profoundly alters the equilibrium predictions of the model. Key comparative statics, such as the response of the equity premium and the risk-free rate to a rise in the IES, switch sign. This means that once we re-estimate the model, the parameters that best fit the data as well as the relative contribution of valuation risk change dramatically. For example, our baseline model with the revised specification requires a coefficient of relative risk aversion (RA) well above the accepted range in the literature.

For intuition, consider the log-stochastic discount factor (SDF) under Epstein-Zin preferences

$$\hat{m}_{t+1} = \theta \log \beta + \theta(\hat{a}_t - \omega \hat{a}_{t+1}) - (\theta/\psi)\Delta \hat{c}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}, \tag{1}$$

where the first, third, and fourth terms—the subjective discount factor (β), log-consumption growth ($\Delta \hat{c}_{t+1}$), and the log-return on the endowment ($\hat{r}_{y,t+1}$)—are all standard in this class of asset pricing models. The second term captures valuation risk, where $\hat{a}_t$ is a time preference shock. In the

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[1]Time preference shocks have been widely used in the macro literature (e.g., Christiano et al. (2011); Eggertsson and Woodford (2003); Justiniano and Primiceri (2008); Rotemberg and Woodford (1997); Smets and Wouters (2003)).

[2]Rapach and Tan (2018) estimate a production asset pricing model with the specification in de Groot et al. (2018).

current literature, $\omega = 0$. Once we revise the preferences and re-derive the log-SDF, we find $\omega = \beta$. When we apply this single alteration to the model, the asset pricing predictions are starkly different.

The asymptote in the current valuation risk specification is related to the preference parameter $\theta \equiv (1 - \gamma)/(1 - 1/\psi)$ that enters the log-SDF, where γ is RA and ψ is the IES. Under constant relative risk aversion (CRRA) preferences, $\gamma = 1/\psi$. In this case, $\theta = 1$ and the log-SDF becomes

$$\hat{m}_{t+1} = \log \beta + (\hat{a}_t - \omega \hat{a}_{t+1}) - \Delta \hat{c}_{t+1}/\psi. \tag{2}$$

The return on the endowment drops out of (1), so the log-SDF is simply composed of the subjective discount factor and consumption growth terms. The advantage of Epstein-Zin preferences is that they decouple γ and ψ, so it is possible to simultaneously have high RA and a high IES. However, there is a nonlinear relationship between θ and ψ, as shown in figure 1. A vertical asymptote occurs at $\psi = 1$: θ tends to infinity as ψ approaches 1 from below while the opposite occurs as ψ approaches 1 from above. When the IES equals 1, θ is undefined. In addition to the vertical asymptote in θ, there is also a horizontal asymptote at $1 - \gamma$ as the IES becomes perfectly elastic.
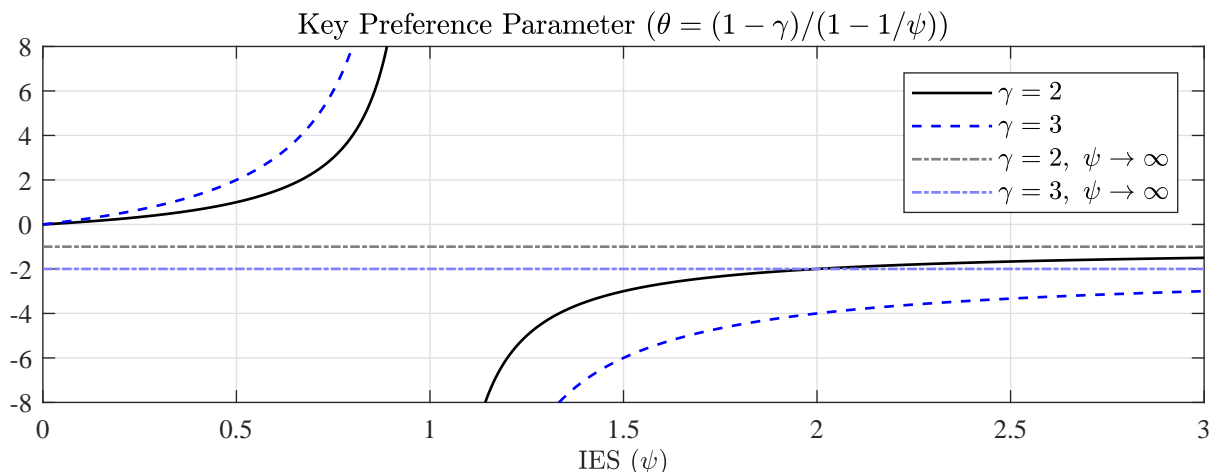
Figure 1: Preference parameter θ in the stochastic discount factor from a model with Epstein-Zin preferences.

Under Epstein and Zin (1989) preferences and the generalization in de Groot et al. (2018) to include valuation risk, the asymptote in figure 1 does not affect asset prices. There is a well-defined equilibrium when the IES equals 1 and asset pricing predictions are robust to small variations in the IES around 1. Continuity is preserved because the weights in the time-aggregator always sum to unity. An alternative interpretation is that the time-aggregator maintains the well-known property that a CES aggregator tends to a Cobb-Douglas aggregator as the elasticity approaches 1. The current specification violates the restriction on the weights so the limiting properties of the CES aggregator break down. As a result, the asymptote in figure 1 permeates key asset pricing moments.

Taken at face value, the asymptote that occurs with the current specification resolves the equity premium (Mehra and Prescott (1985)), risk-free rate (Weil (1989)), and correlation puzzles (Camp-

bell and Cochrane (1999)). Furthermore, when we estimate a model that includes valuation risk and a small long-run predictable component in consumption and dividend growth (henceforth, "long-run risk") following Bansal and Yaron (2004), counterfactual exercises demonstrate that asset prices are almost completely explained by valuation risk, rather than long-run risk. The reason is that valuation risk is able to match the mean equity premium and risk-free rate while maintaining a low correlation between the equity return and consumption and dividend (henceforth, cash flow) growth.

We summarize our main results as follows: (1) The current valuation risk specification fits the data well due to an undesirable asymptote; (2) In our baseline model, the revised specification does not perform as well; (3) When we add Bansal-Yaron long-run risk, revised valuation risk is important for matching the volatility and autocorrelation of the risk-free rate but plays only a minor role in determining most asset pricing moments. Nevertheless, the revised specification fits the data better than the current specification in this model. This is because revised valuation risk has a distinct role, matching the dynamics of the risk-free rate while long-run risk captures the other moments; (4) Extending the model so valuation risk shocks directly affect cash flow growth further improves the empirical fit and helps resolve the correlation puzzle. Conditional on the set of data moments we match, we show this extension is statistically preferred to the addition of stochastic volatility.

The paper proceeds as follows. Section 2 describes the baseline model and the current and revised preference specifications. Section 3 analytically shows why asset prices depend so dramatically on the way valuation risk enters the Epstein-Zin utility function. Section 4 quantifies the effects of the valuation risk specification in our baseline model. Section 5 estimates the relative importance of valuation and long-run risk. Section 6 extends our long-run risk model to include valuation risk shocks to cash flow growth and stochastic volatility on cash flow risk. Section 7 concludes.

2 BASELINE ASSET-PRICING MODEL

We begin by describing our baseline model. Each period t denotes 1 month. There are two assets: an endowment share, $s_{1,t}$, that pays income, y_t, and is in fixed unit supply, and an equity share, $s_{2,t}$, that pays dividends, d_t, and is in zero net supply. The agent chooses $\{c_t, s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$ to maximize

$$U_t^C = [(1-\beta)c_t^{(1-\gamma)/\theta} + a_t^C \beta(E_t[(U_{t+1}^C)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)}, \quad 1 \neq \psi > 0, \tag{3}$$

as used in the current (C) asset pricing literature, or

$$U_t^R = \begin{cases} [(1-a_t^R\beta)c_t^{(1-\gamma)/\theta} + a_t^R\beta(E_t[(U_{t+1}^R)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)}, & \text{for } 1 \neq \psi > 0, \\ c_t^{1-a_t^R\beta}(E_t[(U_{t+1}^R)^{1-\gamma}])^{a_t^R\beta/(1-\gamma)}, & \text{for } \psi = 1, \end{cases} \tag{4}$$

as in the revised (R) specification of de Groot et al. (2018), where E_t is the mathematical expectation operator conditional on information available in period t. The time-preference shocks are

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denoted $a_t^C > 0$ and $0 < a_t^R < 1/\beta$.[3,4] The key difference between the preferences is as follows:

> *The time-varying weights of the time-aggregator in* (3), $(1 - \beta)$ *and* $a_t^C \beta$, *do*
> *not sum to* 1, *whereas the weights in* (4), $(1 - a_t^R \beta)$ *and* $a_t^R \beta$, *do sum to* 1.

The representative agent's choices are constrained by the flow budget constraint given by

$$c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} = (p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}, \tag{5}$$

where $p_{y,t}$ and $p_{d,t}$ are the endowment and dividend claim prices. The optimality conditions imply

$$E_t[m_{t+1}^j r_{y,t+1}] = 1, \quad r_{y,t+1} \equiv (p_{y,t+1} + y_{t+1})/p_{y,t}, \tag{6}$$

$$E_t[m_{t+1}^j r_{d,t+1}] = 1, \quad r_{d,t+1} \equiv (p_{d,t+1} + d_{t+1})/p_{d,t}, \tag{7}$$

where $j \in \{C, R\}$, $r_{y,t+1}$ and $r_{d,t+1}$ are the gross returns on the endowment and dividend claims,

$$m_{t+1}^C \equiv a_t^C \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(V_{t+1}^C)^{1-\gamma}}{E_t[(V_{t+1}^C)^{1-\gamma}]}\right)^{1-\frac{1}{\theta}}, \tag{8}$$

$$m_{t+1}^R \equiv a_t^R \beta \left(\frac{1 - a_{t+1}^R \beta}{1 - a_t^R \beta}\right) \left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi} \left(\frac{(V_{t+1}^R)^{1-\gamma}}{E_t[(V_{t+1}^R)^{1-\gamma}]}\right)^{1-\frac{1}{\theta}}, \tag{9}$$

and V_t^j is the value function that solves the agent's constrained optimization problem.

To permit an approximate analytical solution, we rewrite (6) and (7) as follows

$$E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{y,t+1})] = 1, \tag{10}$$

$$E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{d,t+1})] = 1, \tag{11}$$

where $\hat{m}_{t+1}^j$ is defined in (1) and $\hat{a}_t \equiv \hat{a}_t^C \approx \hat{a}_t^R/(1 - \beta)$ so the shocks in the current and revised models are directly comparable. The common time preference shock, $\hat{a}_{t+1}$, evolves according to

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1}, \ \varepsilon_{a,t+1} \sim \mathbb{N}(0, 1), \tag{12}$$

where $0 \leq \rho_a < 1$ is the persistence of the process, $\sigma_a \geq 0$ is the shock standard deviation, and a hat denotes a log variable. We then apply a Campbell and Shiller (1988) approximation to obtain

$$\hat{r}_{y,t+1} = \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}, \tag{13}$$

$$\hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}, \tag{14}$$

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[3]Kollmann (2016) introduces a time-varying discount factor in an Epstein-Zin setting similar to our formulation. In that setup, the discount factor is a function of endogenously determined consumption rather than a stochastic process.

[4]In the literature, a_t^C typically hits current utility, rather than the risk aggregator. However, with a small change in the timing convention of the preference shock, (3) is isomorphic to the specification used in the literature. We use the specification in (3) because it better facilitates a comparison with the revised preferences. See Appendix A for details.

where $\hat{z}_{y,t+1}$ is the price-endowment ratio, $\hat{z}_{d,t+1}$ is the price-dividend ratio, and

$$\kappa_{y0} \equiv \log(1 + \exp(\hat{z}_y)) - \kappa_{y1}\hat{z}_y, \quad \kappa_{y1} \equiv \exp(\hat{z}_y)/(1 + \exp(\hat{z}_y)), \tag{15}$$

$$\kappa_{d0} \equiv \log(1 + \exp(\hat{z}_d)) - \kappa_{d1}\hat{z}_d, \quad \kappa_{d1} \equiv \exp(\hat{z}_d)/(1 + \exp(\hat{z}_d)), \tag{16}$$

are constants that are functions of the steady-state price-endowment and price-dividend ratios.

To close the model, the processes for log-endowment and log-dividend growth are given by

$$\Delta\hat{y}_{t+1} = \mu_y + \sigma_y\varepsilon_{y,t+1}, \ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{17}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}, \ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{18}$$

where μ_y and μ_d are the steady-state growth rates, $\sigma_y \geq 0$ and $\psi_d\sigma_y \geq 0$ are the shock standard deviations, and π_{dy} captures the covariance between consumption and dividend growth. Asset market clearing implies $s_{1,t} = 1$ and $s_{2,t} = 0$, so the resource constraint is given by $\hat{c}_t = \hat{y}_t$.

Equilibrium includes sequences of quantities $\{\hat{c}_t\}_{t=0}^{\infty}$, prices $\{\hat{m}_{t+1}, \hat{z}_{y,t}, \hat{z}_{d,t}, \hat{r}_{y,t+1}, \hat{r}_{d,t+1}\}_{t=0}^{\infty}$ and exogenous variables $\{\Delta\hat{y}_{t+1}, \Delta\hat{d}_{t+1}, \hat{a}_{t+1}\}_{t=0}^{\infty}$ that satisfy (1), (10)-(14), (17), (18), and the resource constraint, given the state of the economy, $\{\hat{a}_t\}$, and sequences of shocks, $\{\varepsilon_{y,t}, \varepsilon_{d,t}, \varepsilon_{a,t}\}_{t=1}^{\infty}$.

We posit the following solutions for the price-endowment and price-dividend ratios:

$$\hat{z}_{y,t} = \eta_{y0} + \eta_{y1}\hat{a}_t, \quad \hat{z}_{d,t} = \eta_{d0} + \eta_{d1}\hat{a}_t, \tag{19}$$

where $\hat{z}_y = \eta_{y0}$ and $\hat{z}_d = \eta_{d0}$. We solve the model with the method of undetermined coefficients. Appendix B derives the SDF, a Campbell-Shiller approximation, the solution, and key asset prices.

3 INTUITION

This section develops intuition for why the valuation risk specification has such large effects on the model predictions. To simplify the exposition, we consider different stylized shock processes.

3.1 CONVENTIONAL MODEL First, it is useful to review the role of Epstein-Zin preferences and the separation of the RA and IES parameters in matching the risk-free rate and equity premium. For simplicity, we remove valuation risk ($\sigma_a = 0$) and assume endowment/dividend risk is perfectly correlated ($\psi_d = 0$; $\pi_{dy} = 1$). The average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((1/\psi - \gamma)(1 - \gamma) - \gamma^2)\sigma_y^2/2, \tag{20}$$

$$E[ep] = \gamma\sigma_y^2, \tag{21}$$

where the first term in (20) is the subjective discount factor, the second term accounts for endowment growth, and the third term accounts for precautionary savings. Endowment growth creates

an incentive for agents to borrow in order to smooth consumption. Since both assets are in fixed supply, the risk-free rate must be elevated to deter borrowing. When the IES, ψ, is high, agents are willing to accept higher consumption growth so the interest rate required to dissuade borrowing is lower. Therefore, the model requires a fairly high IES to match the low risk-free rate in the data.

With CRRA preferences, higher RA lowers the IES and pushes up the risk-free rate. With Epstein-Zin preferences, these parameters are independent, so a high IES can lower the risk-free rate without lowering RA. Notice the equity premium only depends on RA. Therefore, the model generates a low risk-free rate and modest equity premium with sufficiently high RA and IES parameter values. Of course, there is an upper bound on what constitute reasonable RA and IES values, which is the source of the risk-free rate and equity premium puzzles. Other prominent features such as long-run risk and stochastic volatility à la Bansal and Yaron (2004) help resolve these puzzles.

3.2 VALUATION RISK MODEL   Now consider an example where we remove cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and also assume the time preference shocks are *i.i.d.* ($\rho_a = 0$). Under these assumptions, the assets are identical so $(\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}) = (\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}) \equiv (\kappa_0, \kappa_1, \eta_0, \eta_1)$.

Current Specification   We first solve the model with the current preferences, so the SDF is given by (1) with $\omega = 0$. In this case, the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log \beta + \mu_y/\psi + (\theta - 1)\kappa_1^2 \eta_1^2 \sigma_a^2/2, \tag{22}$$

$$E[ep] = (1 - \theta)\kappa_1^2 \eta_1^2 \sigma_a^2. \tag{23}$$

It is also straightforward to show the log-price-dividend ratio is given by $\hat{z}_t = \eta_0 + \hat{a}_t$ (i.e., the loading on the preference shock, η_1, is 1). Therefore, when the agent becomes more patient and $\hat{a}_t$ rises, the price-dividend ratio rises one-for-one on impact and returns to the stationary equilibrium in the next period. Since η_1 is independent of the IES, there is no endogenous mechanism that prevents the asymptote in θ from influencing the risk-free rate or equity premium. It is easy to see from (16) that $0 < \kappa_1 < 1$. Therefore, θ dominates the average risk-free rate and average equity premium when the IES is near 1. The following result describes the comparative statics with the IES:

> *As ψ approaches 1 from above, θ tends to $-\infty$. As a result, the average risk-free rate tends to $-\infty$ while the average equity premium tends to $+\infty$.*

This key finding illustrates why valuation risk seems like such an attractive feature for resolving the risk-free rate and equity premium puzzles. As the IES tends to 1 from above, θ becomes increasingly negative, which dominates other determinants of the risk-free rate and equity premium. In particular, with an IES slightly above 1, the asymptote in θ causes the average risk-free rate to become arbitrarily small, while making the average equity premium arbitrarily large. Bizarrely, an

IES marginally below 1 (a popular value in the macro literature), generates the opposite predictions. As the IES approaches infinity, $\theta - 1$ tends to γ. Therefore, even when the IES is far above 1, the last term in (22) and (23) is scaled by γ and can still have a meaningful effect on asset prices.

An IES equal to 1 is a key value in the asset pricing literature. For example, it is the basis of the "risk-sensitive" preferences in Hansen and Sargent (2008, section 14.3). Therefore, it is a desirable property for small perturbations around an IES of 1 to not materially alter the predictions of the model. A well-known example of where this property holds is the standard Epstein-Zin asset pricing model without valuation risk. Even though the log-SDF as written in (2) is undefined when the IES equals 1, both the risk-free rate and the equity premium in (20) and (21) are well-defined.

Revised Specification   Next we solve the model with the revised preferences, so the SDF is given by (1) with $\omega = \beta$. In this case, the average risk-free rate and average equity risk premium become

$$E[\hat{r}_f] = -\log \beta + \mu_y/\psi + ((\theta - 1)\kappa_1^2 \eta_1^2 - \theta \beta^2)\sigma_a^2/2, \tag{24}$$

$$E[ep] = ((1 - \theta)\kappa_1 \eta_1 + \theta \beta)\kappa_1 \eta_1 \sigma_a^2. \tag{25}$$

Relative to the current specification, the preference shock loading, η_1, is unchanged. However, both asset prices include a new term that captures the direct effect of valuation risk on current utility, so a rise in a_t that makes the agent more patient raises the value of future certainty equivalent consumption and lowers the value of present consumption. The asymptote occurs with the current specification because it does not account for the effect of valuation risk on current consumption.

With the revised preferences, $\kappa_1 = \beta$ when $\psi = 1$, so the terms involving θ cancel out and the asymptote disappears.[5] The presence of valuation risk lowers the average risk-free rate by $\beta^2 \sigma_a^2/2$ and raises the average equity return by the same amount. Therefore, the average equity premium equals $\beta^2 \sigma_a^2$, which is invariant to the level of RA. When $\psi > 1$, $\kappa_1 > \beta$, so an increase in RA lowers the risk-free rate and raises the equity return. As $\psi \to \infty$, the ratio of the equity premium with revised specification relative to the current specification equals $1 + \beta(1 - \gamma)/(\gamma \kappa_1)$. This means the disparity between the predictions of the two models grows as the level of RA increases.

Expected utility   With CRRA preferences ($\gamma = 1/\psi$), the specifications in (3) and (4) reduce to

$$U_t^C = E_t \sum_{j=0}^{\infty} (1 - \beta)(\textstyle\prod_{i=1}^j a_{t+i-1}^C)\beta^j c_{t+j}^{1-\gamma}/(1 - \gamma),$$

$$U_t^R = E_t \sum_{j=0}^{\infty} (1 - \beta a_{t+j}^R)(\textstyle\prod_{i=1}^j a_{t+i-1}^R)\beta^j c_{t+j}^{1-\gamma}/(1 - \gamma).$$

There is no longer an asymptote with the current preferences because $\theta = 1$ with CRRA utility. The current and revised specifications also generate identical impulse responses to a time preference

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[5]Notice κ_1 is a convolution of the steady-state price-dividend ratio, z_d. When the IES is 1, $z_d = \beta/(1-\beta)$, which is equivalent to its value absent any risk. Therefore, when the IES is 1, valuation risk has no effect on the price-dividend ratio. This result points to a connection with income and substitution effects, which usually cancel when the IES is 1.

shock since $\eta_1 = 1$. However, the two specifications still have different asset pricing implications. Under the current specification, valuation risk has no effect on the risk-free rate and there is no equity premium. With the revised specification, the presence of valuation risk lowers the average risk-free rate by $\beta^2 \sigma_a^2 / 2$ and the average equity premium equals $\beta^2 \sigma_a^2$, just like when the IES equals unity with Epstein-Zin preferences. Therefore, the two expected utility specifications are not interchangeable, but the quantitative differences are insignificant. We can also conclude that the asymptote and stark differences in asset prices between the two Epstein-Zin preference specifications come through the continuation value, V_{t+1}, in the SDF, which drops out with expected utility.
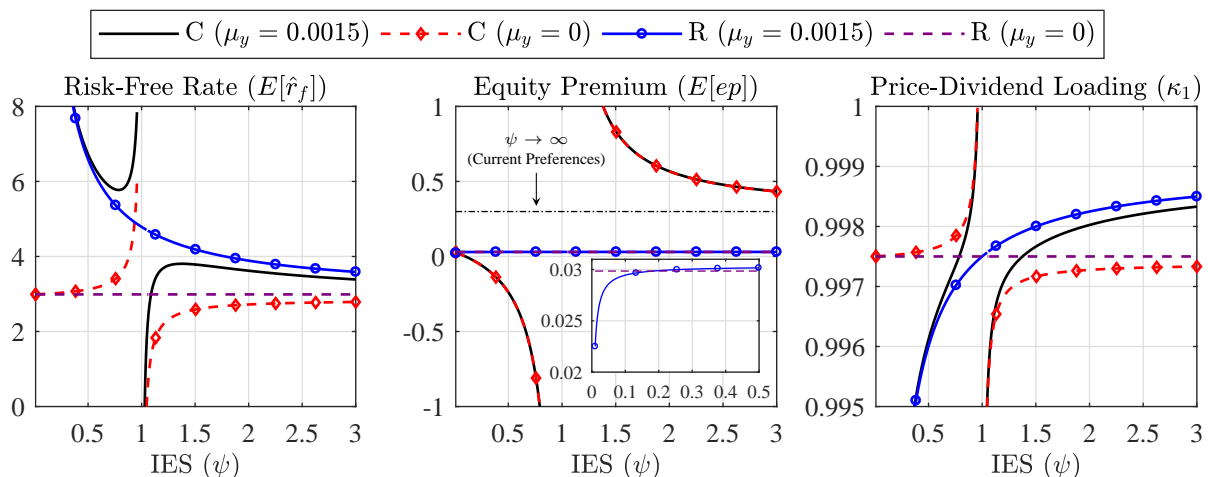

Figure 2: Equilibrium outcomes in the model without cash flow risk ($\sigma_y = 0$; $\mu_y = \mu_d$) and *i.i.d.* preference shocks ($\rho_a = 0$) under the current (C) and revised (R) preference specifications. We set $\beta = 0.9975$, $\gamma = 10$, and $\sigma_a = 0.005$.

3.3 ILLUSTRATION   Our analytical results show the way a time preference shock enters Epstein-Zin utility determines whether the asymptote in θ shows up in equilibrium outcomes. Figure 2 illustrates our results by plotting the average risk-free rate, the average equity premium, and κ_1 (i.e., the marginal response of the price-dividend ratio on the equity return). We focus on the setting in section 3.2 and plot the results under both preferences with and without endowment/dividend growth.

With the current preferences, the average risk-free rate and average equity premium exhibit a vertical asymptote when the IES is 1, regardless of whether μ_y is positive. As a result, the risk-free rate approaches positive infinity as the IES approaches 1 from below and negative infinity as the IES approaches 1 from above. The equity premium has the same comparative statics with the opposite sign, except there is a horizontal asymptote as the IES approaches infinity. These results occur because the current specification misses the direct effect of valuation risk on current consumption.[6]

In contrast, with the revised preferences the average risk-free rate and average equity premium are continuous in the IES, regardless of the value of μ_y. When $\mu_y = 0$, the endowment stream is

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[6]Pohl et al. (2018) find the errors from a Campbell-Shiller approximation of the nonlinear model can significantly affect equilibrium outcomes. Appendix C shows the undesirable asymptote also occurs in the fully nonlinear model.

constant. This means the agent is indifferent about the timing of when the preference uncertainty is resolved, so both κ_1 and the average equity premium are independent of the IES. When $\mu_y > 0$, the agent's incentive to smooth consumption interacts with uncertainty about how (s)he will value the higher future endowment stream.[7] When the IES is large, the agent has a stronger preference for an early resolution of uncertainty, so the equity premium rises as a result of the valuation risk (see the figure 2 inset). Therefore, the qualitative relationship between the IES and the equity premium has different signs under the current and revised specifications. However, the increase in the equity premium is quantitatively small and converges to a level significantly below the value with the current preferences. It is this difference in the sign and magnitude of the relationship between the IES and the equity premium that will explain many of the empirical results in subsequent sections.

4 ESTIMATED BASELINE MODEL

This section returns to the baseline model in section 2, which has both valuation and cash flow risk. We compare the estimates from the model with the current and revised preference specifications.

4.1 DATA AND ESTIMATION METHOD

We construct our data using the procedure in Bansal and Yaron (2004), Beeler and Campbell (2012), Bansal et al. (2016), and Schorfheide et al. (2018). The moments are based on five time series: real per capita consumption expenditures on nondurables and services, the real equity return, real dividends, the real risk-free rate, and the price-dividend ratio. Nominal equity returns are calculated with the CRSP value-weighted return on stocks. We obtain data with and without dividends to back out a time series for nominal dividends. Both series are converted to real using the consumer price index (CPI). The nominal risk-free rate is based on the CRSP yield-to-maturity on 90-day Treasury bills. We first convert the nominal series to real using the CPI. Then we construct an *ex-ante* real rate by regressing the *ex-post* real rate on the nominal rate and inflation over the last year. The consumption data is annual. We convert the monthly asset pricing data to annual series using data from the last month of each year. The model is estimated using annual data from 1929 to 2017—the longest time span available without combining sources.

We estimate each model in two stages. In the first stage, we use Generalized Method of Moments (GMM) to obtain point estimates and a variance-covariance matrix of key moments in the data. In the second stage, we use Simulated Method of Moments (SMM) to search for the parameter vector, θ, that minimizes the squared distance between the GMM point estimates, $\tilde{\Psi}_D$, and median short-sample model moments, $\bar{\Psi}_M$. The weighting matrix, W_D, is the inverse diagonal of the GMM estimate of the variance-covariance matrix, $\tilde{\Sigma}_D$. The objective function, J, is given by

$$J(\theta) = [\bar{\Psi}_M(\theta) - \tilde{\Psi}_D]' W_D [\bar{\Psi}_M(\theta) - \tilde{\Psi}_D]/N_M,$$

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[7]Andreasen and Jørgensen (2018) show how to decouple the agent's timing attitude from the RA and IES values.

where we normalize by the number of moments, N_M, so J reflects the average distance from the moments in $\tilde{\Psi}_D$. We use simulated annealing and then recursively apply Matlab's `fminsearch` to minimize J since gradient-based methods alone did not sufficiently search the parameter space.

Following Albuquerque et al. (2016), our algorithm matches the following 19 moments: the mean and standard deviation of consumption growth, dividend growth, real stock returns, the real risk-free rate, and the price-dividend ratio, the correlation between dividend growth and consumption growth, the correlation between equity returns and both consumption and dividend growth at a 1-, 5-, and 10-year horizon, and the autocorrelation of the price-dividend ratio and real risk-free rate. Appendix D and Appendix E provide more information about our data and estimation method.

Parameter	Current	Revised	Parameter	Current	Revised
γ	1.62617	188.36334	μ_d	0.00190	0.00220
ψ	1.75990	3.53829	ψ_d	4.25999	4.30687
β	0.99807	0.99364	π_{dy}	-0.01606	-0.65893
σ_y	0.00395	0.00378	σ_a	0.00028	0.03198
μ_y	0.00167	0.00171	ρ_a	0.99701	0.99182

(a) Parameter estimates. Current specification: $J = 1.12$; Revised specification: $J = 1.87$.

Moment	Data	Current	Revised	Moment	Data	Current	Revised
$E[\Delta c]$	2.00	2.00	2.05	$SD[\Delta d]$	7.25	5.79	5.66
$E[\Delta d]$	2.06	2.30	2.65	$SD[r_d]$	17.16	17.88	15.62
$E[r_d]$	6.68	5.79	5.72	$SD[r_f]$	2.56	2.82	3.56
$E[r_f]$	0.18	0.18	0.39	$SD[z_d]$	0.44	0.43	0.29
$E[z_d]$	3.47	3.47	3.51	$Corr[\Delta c, \Delta d]$	0.02	0.00	-0.15
$E[ep]$	6.51	5.62	5.33	$AC[r_f]$	0.66	0.95	0.91
$SD[\Delta c]$	1.36	1.36	1.31	$AC[z_d]$	0.92	0.91	0.86

(b) Unconditional short-sample moments given the parameter estimates for each model.

Table 1: Baseline model estimates and asset pricing moments.

4.2 PARAMETER ESTIMATES AND MOMENTS

Table 1 shows the estimated parameter values and selected data and model moments under the current and revised valuation risk specifications.[8] The current estimates are very similar to the values reported in Albuquerque et al. (2016), despite major differences in the data construction. The current model ($J = 1.12$) fits the data better than the revised model ($J = 1.87$). Furthermore, the revised model solved with the current estimates fits the data very poorly ($J = 52.61$), demonstrating that the two specifications yield sharply different quantitative predictions. The current model requires a remarkably low RA value (1.6). The low RA value is due to the asymptote in the current valuation risk specification. An IES close to 1 raises the equity premium to an arbitrarily large extent, while IES values further from 1 cause the equity pre-

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[8]Our data sample effectively begins in 1940 because the long-run correlations shorten our sample by 10 years. Appendix F shows our results are robust to removing the long-run correlations and extending the sample back to 1930.

mium to asymptote at a value much higher than the revised specification generates. Therefore, the current model is able to maintain a very low RA value while matching key asset pricing moments.

The revised model requires extreme parameter values to match the data, similar to a model that only includes transitory cash flow risk. For example, the RA estimate (188.4) is an order of magnitude larger than what is usually accepted in the asset pricing literature.[9] Furthermore, the standard deviation of the preference shock is more than two orders of magnitude larger than the estimate in the current model. Despite these extreme parameter values, the revised model is unable to generate a low enough risk-free rate or a high enough equity premium to match the data. The elevated parameter values also cause the revised model to underpredict the variance of the equity return and overpredict the variance of the risk-free rate. The results demonstrate that valuation risk is not as successful at solving long-standing asset pricing puzzles as the current literature suggests.

5 ESTIMATED LONG-RUN RISK MODEL

In the baseline model, valuation risk explains most of the key asset pricing moments, even after correcting the preference specification. However, the prominent role of valuation risk is not surprising given that we have abstracted from long-run cash flow risk, which is a well-known potential resolution of many asset pricing puzzles. Therefore, this section introduces long-run risk to our baseline model and re-examines the role of valuation risk with both the current and revised preferences.

In order to introduce long-run risk, we modify (17) and (18) as follows:

$$\Delta\hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y\varepsilon_{y,t+1}, \ \varepsilon_{y,t+1} \sim \mathbb{N}(0,1), \tag{26}$$

$$\Delta\hat{d}_{t+1} = \mu_d + \phi_d\hat{x}_t + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}, \ \varepsilon_{d,t+1} \sim \mathbb{N}(0,1), \tag{27}$$

$$\hat{x}_{t+1} = \rho_x\hat{x}_t + \psi_x\sigma_y\varepsilon_{x,t+1}, \ \varepsilon_{x,t+1} \sim \mathbb{N}(0,1), \tag{28}$$

where the specification of the persistent component, $\hat{x}_t$, which is common to both the endowment and dividend growth processes, follows Bansal and Yaron (2004). We apply the same estimation procedure as the baseline model, except we estimate three additional parameters, ϕ_d, ρ_x, and ψ_x.[10]

Table 2 reproduces the results from the baseline model for the model with long-run risk. With the current specification, the presence of long-run risk has almost no effect on the ability of the model to fit the data—the J value is 1.11, compared with 1.12 in the baseline model (despite three new parameters). The parameter estimates are also similar to the baseline model and x_t does not really provide long run risk to cash flows with an estimated persistence parameter, ρ_x, of only 0.49.

Next, we decompose the role of valuation risk and cash flow risk in explaining the asset pricing

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[9]Mehra and Prescott (1985) suggest restricting RA to a maximum of 10. The acceptable range for the IES is less clearly defined in the literature, but values above 3 are atypical. Both revised estimates are well outside of these ranges.

[10]Long-run risk adds one additional state variable, $\hat{x}_t$. Following the guess and verify procedure applied to the baseline model, we use Mathematica to solve for unknown coefficients in the price-endowment and price-dividend ratios.

Parameter	Current	Revised	Parameter	Current	Revised
γ	1.50849	2.82881	π_{dy}	-0.93531	-5.52759
ψ	1.53447	3.95238	σ_a	0.00027	0.01484
β	0.99822	0.99835	ρ_a	0.99707	0.95152
σ_y	0.00382	0.00168	ϕ_d	10.82369	1.71444
μ_y	0.00166	0.00167	ρ_x	0.49798	0.99932
μ_d	0.00188	0.00160	ψ_x	0.15588	0.06208
ψ_d	2.77760	8.14652	$-$	$-$	$-$

(a) Parameter estimates. Current specification: $J = 1.11$; Revised specification: $J = 0.36$.

Moment	Data	Current Specification			Revised Specification		
		All Shocks	Only CFR	Only VR	All Shocks	Only CFR	Only VR
$E[\Delta c]$	2.00	2.00	2.00	2.00	1.99	1.99	2.01
$E[\Delta d]$	2.06	2.28	2.28	2.26	1.91	1.91	1.92
$E[r_d]$	6.68	5.77	3.25	5.84	6.97	6.96	2.48
$E[r_f]$	0.18	0.16	3.42	0.18	0.14	0.28	2.34
$E[z_d]$	3.47	3.47	4.62	3.43	3.47	3.47	5.18
$E[ep]$	6.51	5.61	-0.17	5.66	6.83	6.68	0.13
$SD[\Delta c]$	1.36	1.37	1.37	0.00	1.35	1.35	0.00
$SD[\Delta d]$	7.25	5.67	5.67	0.00	6.13	6.13	0.00
$SD[r_d]$	17.16	17.86	6.14	16.55	17.39	16.82	4.67
$SD[r_f]$	2.56	2.78	0.25	2.77	2.58	0.31	2.56
$SD[z_d]$	0.44	0.43	0.04	0.42	0.47	0.47	0.05
$Corr[\Delta c, \Delta d]$	0.02	0.00	0.00	$-$	0.09	0.09	$-$
$AC[r_f]$	0.66	0.94	0.05	0.95	0.67	0.96	0.66
$AC[z_d]$	0.92	0.91	0.00	0.92	0.93	0.94	0.53

(b) Unconditional short-sample moments given the parameter estimates. All Shocks simulates the model with all of the shocks turned on, Only CFR solves and simulates the model with only the cash flow risk shocks, and Only VR solves and simulates the model with only the valuation risk shocks.

Table 2: Long-run risk model estimates and asset pricing moments.

moments. In addition to showing the estimated moments from the entire model (column entitled All Shocks), table 1b reports the moments from two counterfactual simulations that either remove valuation risk (Only CFR) or cash flow risk (Only VR) from the model. In each case, we re-solve the models after setting $\sigma_a = 0$ for Only CFR and $\sigma_y = 0$ for Only VR, so agents make decisions subject to only one type of risk.[11] Since the asymptote resulting from the current valuation risk specification continues to dominate the determination of asset prices, long run risk plays only a minor role. Valuation risk alone explains almost all of the asset pricing moments, including the near-zero risk free rate and 6.5% equity premium. Without valuation risk, the model generates almost no equity premium, a 3.4% risk-free rate, and equity return volatility much lower than in the data.

The results change dramatically with the revised specification in four key ways. One, the model with long-run risk fits the data much better than the baseline model (the J value falls from 1.87 to 0.36) and the parameter estimates are consistent with Bansal and Yaron (2004). Two, with long-run

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[11]The solution is nonlinear, so the Only CFR and Only VR columns do not have to sum to the All Shocks column.

risk, the revised specification fits the data better than the current specification (with a J value of 0.36 compared to 1.11), in contrast with the results from the baseline model. A way to understand this result is to think of the current specification as competing with long-run risk to explain key asset pricing moments. In contrast, the revised specification complements the original long-run risk model, in that valuation risk is able to match moments that long-run risk struggles to match. Three, RA declines from 188.4 in the baseline model to 2.8 in the model with long-run risk, well within the acceptable range in the literature. Four, the decomposition shows that valuation risk no longer explains the vast majority of asset pricing moments. Cash flow risk by itself generates an equity premium close to the data even though the RA parameter is quite low, whereas valuation risk alone generates almost no equity premium. Instead, valuation risk plays an important role because it explains aspects of the risk-free rate. The standard deviation and autocorrelation of the risk-free rate in the data are 2.6% and 0.66, whereas long-run risk alone generates values of 0.31% and 0.96. We conclude that while valuation risk no longer has the ability to unilaterally resolve long-standing asset pricing puzzles in its revised form, it remains an important aspect of a long-run risk model.

Moment	Data	Current Specification		Revised Specification	
		All Shocks	Only CFR	All Shocks	Only CFR
1-year $Corr[\Delta c, r_d]$	0.01	0.00	0.01	-0.06	-0.06
5-year $Corr[\Delta c, r_d]$	-0.02	0.00	0.01	-0.01	-0.01
10-year $Corr[\Delta c, r_d]$	-0.10	0.01	0.01	0.01	0.02
1-year $Corr[\Delta d, r_d]$	0.15	0.32	0.95	0.30	0.31
5-year $Corr[\Delta d, r_d]$	0.31	0.33	0.99	0.28	0.28
10-year $Corr[\Delta d, r_d]$	0.39	0.33	0.99	0.26	0.27

Table 3: Unconditional short-sample moments given the parameter estimates. All Shocks simulates the model with all of the shocks turned on and Only CFR solves and simulates the model with only the cash flow risk shocks.

The Correlation Puzzle Another important asset pricing puzzle pertains to the correlation between equity returns and fundamentals (Cochrane and Hansen (1992)). In the data, the correlation between equity returns and consumption growth is near zero, regardless of the horizon. The correlation between equity returns and dividend growth is small over short horizons but increases over longer horizons. The central issue is that many asset-pricing models predict too strong of a correlation between stock returns and fundamentals relative to the data. Clearly, if valuation risk generates meaningful volatility in asset returns and yet is uncorrelated with consumption and dividend growth (as in the model in section 2), then valuation risk has the potential to resolve the correlation puzzle.

Table 3 shows the correlations between equity returns and fundamentals over 1-, 5-, and 10-year horizons in the data and the model. We also consider a counterfactual with only cash flow risk (Only CFR). The correlations with consumption growth are similar across the current and revised valuation risk specifications. Consistent with the data, the model predicts a weak correlation over

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all horizons. With both specifications, cash flow risk is sufficient for the model to match the data. The correlations with dividend growth are also similar across the two specifications, but their sources differ. With the current specification, the low correlations are driven by the importance of valuation risk in the model whereas cash flow risk alone overpredicts the correlation. In contrast, cash flow risk plays the primary role with the revised specification. The intuition for these results is straightforward. In a model with long-run risk, most of the volatility in equity returns comes from changes in consumption and dividend growth, while valuation risk is relegated to a secondary role.

6 ESTIMATED EXTENDED LONG-RUN RISK MODELS

This section further examines the role of valuation risk by extending the model with long-run risk and the revised valuation risk specification in two independent ways. First, we consider an extension where valuation risk shocks directly affect consumption and dividend growth, in addition to their effect on asset prices through the SDF (henceforth, the "Demand" shock model). This feature is similar to a discount factor shock in a Dynamic Stochastic General Equilibrium (DSGE) model. For example, in the workhorse New Keynesian model, an increase in the discount factor looks like a typical negative demand shock that lowers interest rates, inflation, and consumption. Therefore, it provides another potential mechanism for valuation risk to help fit the data, especially the correlation moments. Following Albuquerque et al. (2016), we augment (26) and (27) as follows:

$$\Delta \hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon_{y,t+1} + \pi_{ya} \sigma_a \varepsilon_{a,t+1}, \tag{29}$$

$$\Delta \hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_{dy} \sigma_y \varepsilon_{y,t+1} + \psi_d \sigma_y \varepsilon_{d,t+1} + \pi_{da} \sigma_a \varepsilon_{a,t+1}, \tag{30}$$

where π_{ya} and π_{da} determine the covariances between valuation risk shocks and cash flow growth.

Second, we add stochastic volatility to cash flow risk following Bansal and Yaron (2004) (henceforth, the "SV" model). This feature generates a time-varying equity premium and statistically dominates the baseline long-run risk model, as shown by Bansal et al. (2016) (henceforth, BKY). An important question is therefore whether the presence of SV will further diminish the role of valuation risk in its revised specification. To introduce SV, we modify (26)-(28) as follows:

$$\Delta \hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_{y,t} \varepsilon_{y,t+1}, \tag{31}$$

$$\Delta \hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_{dy} \sigma_{y,t} \varepsilon_{y,t+1} + \psi_d \sigma_{y,t} \varepsilon_{d,t+1}, \tag{32}$$

$$\hat{x}_{t+1} = \rho_x \hat{x}_t + \psi_x \sigma_{y,t} \varepsilon_{x,t+1}, \tag{33}$$

$$\sigma_{y,t+1}^2 = \sigma_y^2 + \rho_{\sigma_y}(\sigma_{y,t}^2 - \sigma_y^2) + \nu_y \varepsilon_{\sigma_y,t+1}, \tag{34}$$

where ρ_{σ_y} is the persistence of the process in (34) and ν_y is the standard deviation of the volatility shock. The two models have the exact same number of parameters so the J values are comparable.

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Parameter	Demand	SV	Parameter	Demand	SV
γ	2.817735	3.683705	ρ_a	0.951063	0.951095
ψ	3.398628	3.378736	ϕ_d	1.772998	2.945388
β	0.998433	0.998754	ρ_x	0.999163	0.999143
σ_y	0.000482	0.003335	ψ_x	0.227072	0.018313
μ_y	0.001675	0.001699	π_{ya}	0.082944	–
μ_d	0.001633	0.001695	π_{da}	−1.207079	–
ψ_d	13.683924	4.899773	ρ_σ	–	0.204595
π_{dy}	−4.123352	−0.822328	ν_y	–	0.000001
σ_a	0.015168	0.014923	–	–	–

(a) Parameter estimates. Demand shock model: $J = 0.10$; SV model: $J = 0.36$.

Moment	Data	Demand Shock Model			Stochastic Volatility Model		
		All Shocks	Only CFR	Only VR	All Shocks	Only CFR	Only VR
$E[\Delta c]$	2.00	2.00	1.99	2.01	1.99	1.99	2.04
$E[\Delta d]$	2.06	1.98	1.93	1.98	1.92	1.92	2.03
$E[r_d]$	6.68	6.82	6.78	2.19	6.80	6.78	2.10
$E[r_f]$	0.18	0.17	0.38	2.26	0.42	0.57	1.93
$E[z_d]$	3.47	3.48	3.43	6.16	3.45	3.45	7.30
$E[ep]$	6.51	6.65	6.40	−0.07	6.37	6.21	0.18
$SD[\Delta c]$	1.36	1.36	1.28	0.43	1.37	1.37	0.00
$SD[\Delta d]$	7.25	7.21	3.29	6.32	6.15	6.15	0.00
$SD[r_d]$	17.16	16.68	15.75	3.15	17.43	16.88	4.71
$SD[r_f]$	2.56	2.66	0.37	2.62	2.57	0.22	2.56
$SD[z_d]$	0.44	0.48	0.46	0.03	0.48	0.48	0.05
$Corr[\Delta c, \Delta d]$	0.02	0.01	0.66	–	0.07	0.07	–
$AC[r_f]$	0.66	0.67	0.96	0.66	0.66	0.90	0.66
$AC[z_d]$	0.92	0.94	0.94	0.61	0.93	0.94	0.52

(b) Unconditional short-sample moments given the parameter estimates. All Shocks simulates the model with all of the shocks turned on, Only CFR solves and simulates the model with only the cash flow risk shocks, and Only VR solves and simulates the model with only the valuation risk shocks.

Table 4: Extended long-run risk model estimates and asset pricing moments.

Table 4 shows the parameter estimates and moments for both models with the revised specification.[12] The demand shock model fits the data better than the baseline long-run risk model (the J value declines from 0.36 to 0.10) because it can generate changes in dividend growth independent of consumption growth and cash flow risk. In the baseline long-run risk model, the only way to increase the volatility of dividend growth is through larger cash flow risk shocks. However, a larger shock to consumption growth would have caused the model to over-predict its volatility in the data. Similarly, larger dividend growth shocks, despite helping to improve the fit of dividend growth volatility, would have caused equity return volatility to outstrip the data. In the demand shock model, valuation risk increases the volatility of dividend growth without creating a large effect on equity return volatility because the effect of valuation risk shocks to equity returns are

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[12]In these extended models, the results with the current specification are similar to previous sections. We focus on the revised specification, since previous sections already show the undesirable properties of the current specification.

offset by the response of the price-dividend ratio. These benefits are evident in the counterfactual simulations that isolate the effects of each shock. When the demand shock model only includes valuation risk, there is now sizeable dividend growth volatility (6.3% as compared to 0% in table 2).

The addition of SV has a smaller effect on our estimates. There are four noteworthy results. One, the SV model provides almost no improvement to the empirical fit (the J value declines from 0.364 to 0.356), in contrast with the demand shock model. Two, the estimates of the valuation risk persistence (ρ_a) and standard deviation (σ_a) are roughly the same in the models with and without SV. This suggests the presence of SV does not diminish the role of valuation risk. Three, the persistence (ρ_σ) and standard deviation (ν_y) of the SV process are relatively small, further indicating SV does not play a major role in matching these moments. Four, the counterfactuals show that with only cash flow risk, the SV model continues to under-predict the volatility of the risk-free rate.

The low RA and limited role of SV may seem surprising in light of the results in BKY. We attribute the differences to three factors. One, we match different moments. Our estimation includes correlations between cash flow growth and equity returns as well as the volatility and autocorrelation of the risk-free rate, whereas BKY include higher order moments such as the heteroscedasticity of consumption. Two, our effective sample excludes the Great Depression. Our raw data starts in 1929 as in BKY, but we lose 10 years since we match long-run correlations and use a balanced sample. Third, we include valuation risk in our model, which is an additional source of volatility.[13]

Moment	Data	Demand Shock Specification		Stochastic Volatility Specification	
		All Shocks	Only CFR	All Shocks	Only CFR
1-year $Corr[\Delta c, r_d]$	0.01	−0.01	0.04	−0.04	−0.04
5-year $Corr[\Delta c, r_d]$	−0.02	0.01	0.08	−0.02	−0.02
10-year $Corr[\Delta c, r_d]$	−0.10	0.04	0.10	0.00	0.00
1-year $Corr[\Delta d, r_d]$	0.15	0.18	0.13	0.31	0.31
5-year $Corr[\Delta d, r_d]$	0.31	0.27	0.13	0.27	0.27
10-year $Corr[\Delta d, r_d]$	0.39	0.30	0.14	0.26	0.26

Table 5: Unconditional short-sample moments given the parameter estimates. All Shocks simulates the model with all of the shocks turned on and Only CFR solves and simulates the model with only the cash flow risk shocks.

The Correlation Puzzle Table 5 shows the demand shock model also makes progress in solving the correlation puzzle. Just like in the long-run risk model in section 5, both of the extended models predict near-zero correlations between consumption growth and equity returns over a 1-, 5-, and 10-year horizon. However, in the demand shock model, the correlations counterfactually strengthen over 5- and 10-year horizons when it only includes cash flow risk. The clearest advantage of the demand shock model is its ability to match the correlations between equity returns and dividend

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[13]There are also differences in the weighting matrix (BKY recursively update it based on model estimates, rather than fixing it to data) and in how the moments are calculated (BKY use theoretical instead of short-sample moments).

growth. Specifically, it predicts a weak correlation at a 1-year horizon and a stronger correlation over a 5-year horizon. In the SV model, the opposite result occurs. Furthermore, the counterfactual simulations show that valuation risk is crucial for obtaining a weak correlation at a 1-year horizon. These results emphasize that the data strongly prefers the demand shock model with correlated cash flow risk and revised valuation risk over the more traditional long-run risk model with SV.

7 CONCLUSION

The way valuation risk enters Epstein-Zin recursive utility has important implications. Under the current specification in the literature, an undesirable asymptote in the parameter space permeates equilibrium outcomes. The asymptote occurs as the IES approaches unity, but it profoundly affects asset prices even when the IES is well above one. As a consequence, the asymptote perversely allows valuation risk alone to explain the historically low risk-free rate and high equity premium.

Once we revise the preference specification to remove the undesirable asymptote, valuation risk has a much smaller role in explaining asset pricing moments. In particular, it is no longer able to unilaterally resolve the equity premium, risk-free rate, and correlation puzzles. However, we show that valuation risk still plays an important role in matching the volatility and autocorrelation of the risk-free rate. Furthermore, allowing valuation risk shocks to directly affect cash flow growth introduces an important source of volatility to the model that significantly improves the empirical fit and helps resolve the correlation puzzle. We conclude that valuation risk is not as important as the current literature suggests, but it still has a consequential role in explaining certain asset prices.

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A ISOMORPHIC REPRESENTATIONS OF THE CURRENT SPECIFICATION

In the current literature, the preference shock typically hits current utility. If, for simplicity, we abstract from Epstein-Zin preferences, then the value function and Euler equation are given by

$$V_t = \alpha_t u(c_t) + \beta E_t[V_{t+1}], \tag{35}$$

$$\beta E_t[(\alpha_{t+1}/\alpha_t)u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{36}$$

The shock follows $\Delta\hat{\alpha}_{t+1} = \rho\Delta\hat{\alpha}_t + \sigma_\alpha\varepsilon_t$, so the change in α_t is known at time t. Alternatively, if the preference shock hits future consumption, the value function and Euler equation are given by

$$V_t = u(c_t) + a_t\beta E_t[V_{t+1}], \tag{37}$$

$$a_t\beta E_t[u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{38}$$

If the shock follows $\hat{a}_t = \rho\hat{a}_{t-1} + \sigma_a\varepsilon_t$, the two specifications are isomorphic because setting $a_t \equiv \alpha_{t+1}/\alpha_t$ in (38) yields (36). We use the second specification because it is easier to compare the current and revised preferences when the shock always shows up in the Euler equation in levels.

B ANALYTICAL DERIVATIONS

Stochastic Discount Factor The value function for specification $j \in \{C, R\}$ is given by

$$V_t^j = \max[w_{1,t}^j c_t^{(1-\gamma)/\theta} + w_{2,t}^j (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta}]^{\theta/(1-\gamma)}$$
$$- \lambda_t(c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} - (p_{y,t} + y_t)s_{1,t-1} - (p_{d,t} + d_t)s_{2,t-1}),$$

where $w_{1,t}^C = 1-\beta$, $w_{1,t}^R = 1-a_t^R\beta$, $w_{2,t}^C = a_t^C\beta$, and $w_{2,t}^R = a_t^R\beta$. The optimality conditions imply

$$w_{1,t}^j(V_t^j)^{1/\psi}c_t^{-1/\psi} = \lambda_t, \tag{39}$$

$$w_{2,t}^j(V_t^j)^{1/\psi}(E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta-1}E_t[(V_{t+1}^j)^{-\gamma}(\partial V_{t+1}^j/\partial s_{1,t})] = \lambda_t p_{y,t}, \tag{40}$$

$$w_{2,t}^j(V_t^j)^{1/\psi}(E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta-1}E_t[(V_{t+1}^j)^{-\gamma}(\partial V_{t+1}^j/\partial s_{2,t})] = \lambda_t p_{d,t}, \tag{41}$$

where $\partial V_t^j/\partial s_{1,t-1} = \lambda_t(p_{y,t} + y_t)$ and $\partial V_t^j/\partial s_{2,t-1} = \lambda_t(p_{d,t} + d_t)$ by the envelope theorem. Updating the envelope conditions and combining (39)-(41) generates (8) and (9) in the main text.

Following Epstein and Zin (1991), we posit the following minimum state variable solution:

$$V_t^j = \xi_{1,t}s_{1,t-1} + \xi_{2,t}s_{2,t-1} \quad \text{and} \quad c_t = \xi_{3,t}s_{1,t-1} + \xi_{4,t}s_{2,t-1}. \tag{42}$$

where ξ is a vector of unknown coefficients. The envelope conditions combined with (39) imply

$$\xi_{1,t} = w_{1,t}^j(V_t^j)^{1/\psi}c_t^{-1/\psi}(p_{y,t} + y_t), \tag{43}$$

$$\xi_{2,t} = w_{1,t}^j(V_t^j)^{1/\psi}c_t^{-1/\psi}(p_{d,t} + d_t). \tag{44}$$

Multiplying the respective conditions by $s_{1,t-1}$ and $s_{2,t-1}$ and then adding yields

$$V_t^j = w_{1,t}^j(V_t^j)^{1/\psi}c_t^{-1/\psi}((p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}), \tag{45}$$

which after plugging in the budget constraint, (5), and imposing equilibrium can be written as

$$(V_t^j)^{(1-\gamma)/\theta} = w_{1,t}^j c_t^{-1/\psi}(c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t}) = w_{1,t}^j c_t^{-1/\psi}(c_t + p_{y,t}). \tag{46}$$

Therefore, the optimal value function is given by

$$w_{1,t}^j c_t^{-1/\psi}p_{y,t} = w_{2,t}^j(E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta}. \tag{47}$$

Solving (46) for V_t^j and (47) for $E_t[(V_{t+1}^j)^{1-\gamma}]$ and then plugging into (8) and (9) implies

$$m_{t+1}^j = (x_t^j)^\theta(c_{t+1}/c_t)^{-\theta/\psi}r_{y,t+1}^{\theta-1}, \tag{48}$$

where $x_t^j \equiv w_{2t}^j w_{1t+1}^j/w_{1t}^j$. Taking logs of (48) yields (1), given the following definitions:

$$\hat{x}_t^C = \hat{\beta} + \hat{a}_t^C,$$

$$\hat{x}_t^R = \hat{\beta} + \hat{a}_t^R + \log(1 - \beta\exp(\hat{a}_{t+1}^R)) - \log(1 - \beta\exp(\hat{a}_t^R)) \approx \hat{\beta} + (\hat{a}_t^R - \beta\hat{a}_{t+1}^R)/(1 - \beta),$$

and $\hat{a}_t \equiv \hat{a}_t^C = \hat{a}_t^R/(1 - \beta)$ so the preference shocks in the current and revised models are directly comparable. It immediately follows that $\hat{x}_t^j = \hat{\beta} + \hat{a}_t - \omega^j\hat{a}_{t+1}$ as in (1), where $\omega^C = 0$ and $\omega^R = \beta$.

Campbell-Shiller Approximation The return on the endowment is approximated by

$$
\begin{aligned}
\hat{r}_{y,t+1} &= \log(y_{t+1}(p_{y,t+1}/y_{t+1}) + y_{t+1}) - \log(y_t(p_{y,t}/y_t)) \\
&= \log(\exp(\hat{z}_{y,t+1}) + 1) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&\approx \log(\exp(\hat{z}_y) + 1) + \exp(\hat{z}_y)(\hat{z}_{y,t+1} - \hat{z}_y)/(1 + \exp(\hat{z}_y)) - \hat{z}_{y,t} + \Delta\hat{y}_{t+1} \\
&= \kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta\hat{y}_{t+1}.
\end{aligned}
$$

The derivation for the equity return, $\hat{r}_{d,t+1}$, is analogous to the return on the endowment.

20

Model Solution We use a guess and verify method. For the endowment claim, we obtain

$$
\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{y,t+1})]) \\
&= \log(E_t[\exp(\theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + \theta(1 - 1/\psi)\Delta\hat{y}_{t+1} + \theta(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}))]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + \theta(1 - 1/\psi)(\mu_y + \sigma_y\varepsilon_{y,t+1}) \\ +\theta\kappa_{y0} + \theta\kappa_{y1}(\eta_{y0} + \eta_{y1}\hat{a}_{t+1}) - \theta(\eta_{y0} + \eta_{y1}\hat{a}_t) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(1 - 1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) \\ +\theta(1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\ +\theta(1 - 1/\psi)\sigma_y\varepsilon_{y,t+1} + \theta(\kappa_{y1}\eta_{y1} - \omega^j)\sigma_a\varepsilon_{a,t+1} \end{array}\right)\right]\right) \\
&= \theta\hat{\beta} + \theta(1 - 1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta^2}{2}(1 - 1/\psi)^2\sigma_y^2 \\
&\quad + \tfrac{\theta^2}{2}(\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2 + \theta(1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t,
\end{aligned}
$$

where the last equality follows from the log-normality of $\exp(\varepsilon_{y,t+1})$ and $\exp(\varepsilon_{a,t+1})$.

After equating coefficients, we obtain the following exclusion restrictions:

$$\hat{\beta} + (1 - 1/\psi)\mu_y + (\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta}{2}((1 - 1/\psi)^2\sigma_y^2 + (\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2) = 0, \quad (49)$$

$$1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1) = 0. \quad (50)$$

For the dividend claim, we obtain

$$
\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{d,t+1})]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + (\theta(1 - 1/\psi) - 1)\Delta\hat{y}_{t+1} + \Delta\hat{d}_{t+1} \\ +(\theta - 1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) + (\kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t}) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + (\theta(1 - 1/\psi) - 1)\mu_y + \mu_d \\ +(\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\ +(\theta(1 - \omega^j\rho_a) + (\theta - 1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\ (\pi_{dy} - \gamma)\sigma_y\varepsilon_{y,t+1} + ((\theta - 1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1} \end{array}\right)\right]\right) \\
&= \theta\hat{\beta} + (\theta(1 - 1/\psi) - 1)\mu_y + \mu_d + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
&\quad + (\theta(1 - \omega^j\rho_a) + (\theta - 1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\
&\quad + \tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta - 1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2).
\end{aligned}
$$

Once again, equating coefficients implies the following exclusion restrictions:

$$\theta\hat{\beta} + (\theta(1 - 1/\psi) - 1)\mu_y + \mu_d + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1))$$

$$+\tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta - 1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2) = 0, \quad (51)$$

$$\theta(1 - \omega^j\rho_a) + (\theta - 1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1) = 0. \quad (52)$$

Equations (49)-(52), along with (15) and (16), form a system of 8 equations and 8 unknowns.

Asset Prices Given the coefficients, we can solve for the risk free rate. The Euler equation implies

$$\hat{r}_{f,t} = -\log(E_t[\exp(\hat{m}_{t+1})]) = -E_t[\hat{m}_{t+1}] - \tfrac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}],$$

since the risk-free rate is known at time-t. The pricing kernel is given by

$$\begin{aligned}
\hat{m}_{t+1} &= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{y}_{t+1} + (\theta - 1)\hat{r}_{y,t+1}\\
&= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - \gamma\Delta\hat{y}_{t+1} + (\theta - 1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t})\\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\theta(1 - \omega^j) + (\theta - 1)\eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t\\
&\quad + ((\theta - 1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} - \gamma\sigma_y\varepsilon_{y,t+1}\\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (1 - \omega^j\rho_a)\hat{a}_t\\
&\quad + ((\theta - 1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1}) - \gamma\sigma_y\varepsilon_{y,t+1},
\end{aligned}$$

where the last line follows from imposing (50). Therefore, the risk-free rate is given by

$$\begin{aligned}
\hat{r}_{f,t} &= \gamma\mu_y - \theta\hat{\beta} - (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) - (1 - \omega^j\rho_a)\hat{a}_t\\
&\quad - \tfrac{1}{2}\gamma^2\sigma_y^2 - \tfrac{1}{2}((\theta - 1)\kappa_{y1}\eta_{y1} - \theta\omega^j)^2\sigma_a^2.
\end{aligned}$$

Note that $\hat{r}_{f,t} = \log(E_t[\exp(\hat{r}_{f,t})])$. After plugging in (49), we obtain

$$\hat{r}_{f,t} = \mu_y/\psi - \hat{\beta} - (1 - \omega^j\rho_a)\hat{a}_t + \tfrac{1}{2}((\theta - 1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi - \gamma)(1 - \gamma) - \gamma^2)\sigma_y^2.$$

Therefore, the unconditional expected risk-free rate is given by

$$E[\hat{r}_f] = -\hat{\beta} + \mu_y/\psi + \tfrac{1}{2}((\theta - 1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi - \gamma)(1 - \gamma) - \gamma^2)\sigma_y^2. \tag{53}$$

We can also derive an expression for the equity premium, $E_t[ep_{t+1}]$, which given by

$$\log(E_t[\exp(\hat{r}_{d,t+1} - \hat{r}_{f,t})]) = E_t[\hat{r}_{d,t+1}] - \hat{r}_{f,t} + \tfrac{1}{2}\operatorname{Var}_t[\hat{r}_{d,t+1}] = -\operatorname{Cov}_t[\hat{m}_{t+1}, \hat{r}_{d,t+1}],$$

where the last equality stems from the Euler equation, $E_t[\hat{m}_{t+1} + \hat{r}_{d,t+1}] + \tfrac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1} + \hat{r}_{d,t+1}] = 0$. We already solved for the SDF, so the last step is to solve for the equity return, which given by

$$\begin{aligned}
\hat{r}_{d,t+1} &= \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}\\
&= \kappa_{d0} + \kappa_{d1}(\eta_{d0} + \eta_{d1}\hat{a}_{t+1}) - (\eta_{d0} + \eta_{d1}\hat{a}_t) + \Delta\hat{d}_{t+1}\\
&= \mu_d + \kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1)\hat{a}_t + \kappa_{d1}\eta_{d1}\sigma_a\varepsilon_{a,t+1} + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}.
\end{aligned}$$

Therefore, the unconditional equity premium can be written as

$$E[ep] = \gamma\pi_{dy}\sigma_y^2 + (\theta\omega^j + (1 - \theta)\kappa_{y1}\eta_{y1})\kappa_{d1}\eta_{d1}\sigma_a^2. \tag{54}$$

B.1 SPECIAL CASE 1 ($\sigma_a = \psi_d = 0$ & $\pi_{dy} = 1$) In this case, there is no valuation risk ($\hat{a}_t = 0$) and cash flow risk is perfectly correlated ($\Delta\hat{y}_{t+1} = \mu_y + \sigma_y \varepsilon_{y,t+1}$; $\Delta\hat{d}_{t+1} = \mu_d + \sigma_y \varepsilon_{y,t+1}$). Under these two assumptions, it is easy to see that (53) and (54) reduce to (20) and (21) in the main text.

B.2 SPECIAL CASE 2 ($\sigma_y = 0$, $\rho_a = 0$, & $\mu_y = \mu_d$) In this case, there is no cash flow risk ($\Delta\hat{y}_{t+1} = \Delta\hat{d}_{t+1} = \mu_y$) and the time preference shocks are *i.i.d.* ($\hat{a}_{t+1} = \sigma_a \varepsilon_{a,t+1}$). Under these two assumptions, the return on the endowment and dividend claims are identical, so $\{\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}\} = \{\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}\} \equiv \{\kappa_0, \kappa_1, \eta_0, \eta_1\}$. Therefore, (53) and (54) reduce to (22) and (23) for the current specification and (24) and (25) for the revised specification.

The exclusion restriction, (50), implies $\eta_1 = 1$ so (49) simplifies to

$$0 = \hat{\beta} + (1 - 1/\psi)\mu_y + \kappa_0 + \eta_0(\kappa_1 - 1) + \tfrac{\theta}{2}(\kappa_1 - \omega^j)^2 \sigma_a^2. \tag{55}$$

First, recall that $0 < \kappa_1 < 1$. Therefore, the asymptote in θ will permeate the solution with the current preferences ($\omega^C = 0$). However, with the revised preferences ($\omega^R = \beta$), we guess and verify that $\kappa_1 = \beta$ when $\psi = 1$. In this case, (55) reduces to $\hat{\beta} + \kappa_0 + \eta_0(\beta - 1) = 0$. Combining with (15), this restriction implies that $\eta_0 = \log\beta - \log(1 - \beta)$ and $\kappa_0 = -(1 - \beta)\log(1 - \beta) - \beta\log\beta$. Plugging the expressions for η_0, κ_0, and κ_1 back into (15) and (55) verifies our initial guess for κ_1.

C NONLINEAR MODEL ASYMPTOTE

The Euler equation, written in terms of the price-dividend ratio, is given by

$$z_t = \frac{a_t\beta}{1 - \chi^j a_t \beta} \left(E_t \left[\underbrace{((1 - \chi^j a_{t+1}\beta)\mu_{t+1}^{1-1/\psi}(1 + z_{t+1}))^\theta}_{x_{t+1}} \right] \right)^{1/\theta}, \tag{56}$$

assuming $\mu_{t+1} \equiv y_{t+1}/y_t = d_{t+1}/d_t$. Notice the asymptote disappears if $\mathrm{SD}(x_{t+1}) \to 0$ as $\psi \to 1$.

Consider first the case without valuation risk, so $a_t = 1$ for all t. The Euler equation reduces to

$$z_t = \beta(E_t[(\mu_{t+1}^{1-1/\psi}(1 + z_{t+1}))^\theta])^{1/\theta}. \tag{57}$$

When $\psi = 1$, we guess and verify that $z_t = \beta/(1 - \beta)$, so the price-dividend ratio is constant. This is the well know result that when the IES is 1, the income and substitution effects of a change in endowment growth offset. Therefore, the price-dividend ratio does not respond to cash flow risk.

Consider next the case when a_t is stochastic under the revised preferences ($\chi^R = 1$). In this case, when $\psi = 1$ we guess and verify that $z_t = a_t\beta/(1 - a_t\beta)$. Notice the price dividend ratio is time-varying but independent of θ. Therefore, an asymptote does not affect equilibrium outcomes.

Finally, consider what happens under the current preferences ($\chi^C = 0$), which do not account

for the offsetting movements in $1 - a_t\beta$. To obtain a closed-form solution for any IES, we assume $\mu_t = \mu$ and the preference shock evolves according to $\log(1 + a_{t+1}\eta) = \sigma\varepsilon_{t+1}$, where ε_{t+1} is standard normal. Under these assumptions, we guess and verify that the price-dividend ratio is given by

$$z_t = a_t\eta = a_t\beta\mu^{1-1/\psi}\exp(\theta\sigma^2/2). \tag{58}$$

In this case, θ appears in the price-dividend ratio, so the asymptote affects equilibrium outcomes. These results prove that the asymptote is not due to a Campbell-Shiller approximation of the model.

D DATA SOURCES

We drew from the following data sources to estimate our models:

1. **[$RCONS$] Per Capita Real PCE (excluding durables)**: Annual, chained 2012 dollars. Source: Bureau of Economic Analysis, National Income and Product Accounts, Table 7.1.

2. **[$RETD$] Value-Weighted Return (including dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETD).

3. **[$RETX$] Value-Weighted Return (excluding dividends)**: Monthly. Source: Wharton Research Data Services, CRSP Stock Market Indexes (CRSP ID: VWRETX).

4. **[CPI] Consumer Price Index for All Urban Consumers**: Monthly, not seasonally adjusted, index 1982-1984=100. Source: Bureau of Labor Statistics (FRED ID: CPIAUCNS).

5. **[RFR] Risk-free Rate**: Monthly, annualized yield calculated from nominal price. Source: Wharton Research Data Services, CRSP Treasuries, Risk-free Series (CRSP ID: TMYTM).

We applied the following transformations to the above data sources:

1. **Annual Per Capita Real Consumption Growth (annual frequency)**:

$$\Delta\hat{c}_t = 100\log(RCONS_t/RCONS_{t-1})$$

2. **Annual Real Dividend Growth (monthly frequency)**:

$$P_{1928M1} = 100, \quad P_t = P_{t-1}(1 + RETX_t), \quad D_t = (RETD_t - RETX_t)P_{t-1},$$
$$d_t = \sum_{i=t-11}^{t}D_i/CPI_t, \quad \Delta\hat{d}_t = 100\log(d_t/d_{t-12})$$

3. **Annual Real Equity Return (monthly frequency)**:

$$\pi_t^m = \log(CPI_t/CPI_{t-1}), \quad \hat{r}_{d,t} = 100\sum_{i=t-11}^{t}(\log(1 + RETD_i) - \pi_i^m)$$

4. **Annual Real Risk-free Rate (monthly frequency)**:

$$rfr_t = RFR_t - \log(CPI_{t+3}/CPI_t), \quad \pi_t^q = \log(CPI_t/CPI_{t-12})/4,$$
$$\hat{r}_{f,t} = 400(\hat{\beta}_0 + \hat{\beta}_1 RFR_t + \hat{\beta}_2 \pi_t^q),$$

where $\hat{\beta}_j$ are the OLS estimates in a regression of the *ex-post* real rate, rfr, on the nominal rate, RFR, and lagged inflation, π^q. The fitted values are estimates of the *ex-ante* real rate.

5. **Price-Dividend Ratio (monthly frequency)**:

$$\hat{z}_{d,t} = \log(P_t/\textstyle\sum_{i=t-11}^t D_i)$$

We use December of each year to convert each of the monthly time series to an annual frequency.

E ESTIMATION METHOD

The estimation method is conducted in two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 10 lags. The second stage implements a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The following steps outline the algorithm:

1. Use GMM to estimate the data moments, $\tilde{\Psi}_D$, and variance-covariance matrix, $\tilde{\Sigma}_D$.

2. Specify a guess, $\hat{\theta}_0$, for the N_e estimated parameters and the parameter variance-covariance matrix, Σ_P, which is initialized as a diagonal matrix. Note that θ is model dependent.

3. Use simulated annealing to minimize the distance between the data and model moments.

   (a) For all $i \in \{0, \dots, N_d\}$, perform the following steps:

   i. Draw a candidate vector of parameters, $\hat{\theta}_i^{cand}$, where

   $$\hat{\theta}_i^{cand} \sim \begin{cases} \hat{\theta}_0 & \text{for } i = 0, \\ \mathbb{N}(\hat{\theta}_{i-1}, c\Sigma_P) & \text{for } i > 0. \end{cases}$$

   We set c to target an acceptance rate of roughly 30%. For the revised specification, we impose a restriction on $\hat{\theta}_i^{cand}$ such that $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$, so the utility function weights are positive in 99.997% of the simulated observations.

   ii. Solve the Campbell-Shiller approximation of the model given $\hat{\theta}_i^{cand}$.

iii. Simulate the monthly model $1,000$ times for the same length as the data. We draw initial states, $\hat{a}_0$, from $\mathbb{N}(0, \sigma_a^2/(1 - \rho_a^2))$. For each simulation j, calculate the moments, $\Psi_{M,j}(\hat{\theta}_i^{cand})$, analogous to those in the data.

iv. Calculate the median moments across the short-sample simulations, $\bar{\Psi}_M(\hat{\theta}_i^{cand}) = \text{median}\left(\{\Psi_{M,j}(\hat{\theta}_i^{cand})\}_{j=1}^{1000}\right)$, and evaluate the objective function given by

$$J_i^{cand} = [\bar{\Psi}_M(\hat{\theta}_i^{cand}) - \tilde{\Psi}_D]'W_D[\bar{\Psi}_M(\hat{\theta}_i^{cand}) - \tilde{\Psi}_D]/N_M,$$

where W_D is the inverse diagonal of the GMM estimate of the matrix, $\tilde{\Sigma}_D$.

v. Accept or reject the candidate draw according to

$$(\hat{\theta}_i, J_i) = \begin{cases} (\hat{\theta}_i^{cand}, J_i^{cand}) & \text{if } i = 0, \\ (\hat{\theta}_i^{cand}, J_i^{cand}) & \text{if } \min(1, \exp(J_{i-1} - J_i^{cand})/t) > \hat{u}, \\ (\hat{\theta}_{i-1}, J_{i-1}) & \text{otherwise}, \end{cases}$$

where t is the temperature and $\hat{u}$ is a draw from a uniform distribution. The lower the temperature, the more likely it is that the candidate draw is rejected.

(b) Find the parameter draw $\hat{\theta}^{min}$ that corresponds to J^{min}, and update Σ_P.

i. Discard the first $N_d/2$ draws. Stack the remaining draws in a $N_d/2 \times N_e$ matrix, $\hat{\Theta}$, and define $\tilde{\Theta} = \hat{\Theta} - \sum_{i=1}^{N_d/2} \hat{\theta}_{i,j}/(N_d/2)$.

ii. Calculate $\Sigma_P^{up} = \tilde{\Theta}'\tilde{\Theta}/(N_d/2)$.

4. Repeat the previous step N_{SMM} times, initializing at draw $\hat{\theta}_0 = \hat{\theta}^{min}$ and covariance matrix $\Sigma_P = \Sigma_P^{up}$. Gradually decrease the temperature each time. Of all the draws, find the lowest 20 J values, denoted $\{J_j^{guess}\}_{j=1}^{20}$, and the corresponding parameter draws, $\{\theta_j^{guess}\}_{j=1}^{20}$.

5. Run Matlab's `fminsearch`, using $\{\theta_j^{guess}\}_{j=1}^{20}$ as an initial guesses. We simulated the model $5,000$ times on each iteration and set the tolerance on θ to 0.01. The resulting minimum is $\hat{\theta}_j^{min}$ and the corresponding J value is J_j^{min}. Repeat, each time updating the guess, until $J_j^{guess} - J_j^{min} < 0.001$. The final parameter estimates correspond to the $\min\{J_j^{min}\}_{j=1}^{20}$.

F ROBUSTNESS OF THE BASELINE MODEL ESTIMATES

The estimation procedure that generates the results in the main paper matches long-run correlations between equity returns and cash flow growth. We decided to include these moments for two reasons. One, they are used in Albuquerque et al. (2016), who estimate similar asset pricing models. Two, it allows us to re-examine whether valuation risk helps resolve the correlation puzzle. However, there is one main drawback of matching long-run correlations—it forces us to remove

Parameter	Current	Revised	Parameter	Current	Revised
γ	1.32319	65.25786	μ_d	0.00153	0.00230
ψ	1.57352	5.05068	ψ_d	1.46282	0.96961
β	0.99806	0.99538	π_{dy}	0.80217	0.40932
σ_y	0.00579	0.00567	σ_a	0.00032	0.03518
μ_y	0.00158	0.00159	ρ_a	0.99672	0.98999

(a) Parameter estimates. Current specification: $J = 1.89$; Revised specification: $J = 3.23$.

Moment	Data	Current	Revised	Moment	Data	Current	Revised
$E[\Delta c]$	1.89	1.89	1.91	$SD[\Delta d]$	11.09	3.32	2.05
$E[\Delta d]$	1.47	1.84	2.77	$SD[r_d]$	19.15	18.70	13.75
$E[r_d]$	6.51	5.47	5.93	$SD[r_f]$	2.72	3.18	3.66
$E[r_f]$	0.25	0.24	0.23	$SD[z_d]$	0.45	0.45	0.25
$E[z_d]$	3.42	3.44	3.47	$Corr[\Delta c, \Delta d]$	0.54	0.48	0.39
$E[ep]$	6.26	5.23	5.70	$AC[r_f]$	0.68	0.94	0.89
$SD[\Delta c]$	1.99	2.00	1.96	$AC[z_d]$	0.89	0.91	0.85

(b) Unconditional short-sample moments given the parameter estimates for each model.

Table 6: Baseline model estimates and moments without matching long-run correlations and a longer data sample.

the Great Depression period to maintain a balanced sample. For example, since we include the correlation between equity returns and consumption growth over the last 10 years, our effective sample runs from 1940 to 2017, even though our raw data starts in 1929 (i.e., the first growth rate is in 1930). Therefore, the decision of whether to include these long-run correlations changes some of the other moments we are trying to match. One major change is to the standard deviation of dividend growth, which increases from 7.25 to 11.09. The correlation between consumption and dividend growth is also much stronger, rising from 0.02 to 0.54. This section tests the robustness of the estimates from our baseline model by removing the long-run correlations and extending the sample.

Table 6 shows the parameter estimates and moments for the baseline model with the longer sample. Our qualitative results are unchanged, despite the differences in the data moments. The current specification fits the data very well with small RA and IES values and the results are driven by valuation risk. In contrast, the revised specification fits the data worse (the J value rises from 1.89 to 3.23), the RA value is well outside the accepted range in the literature, and the preference shock standard deviation is two orders of magnitude larger than it is with the current specification.