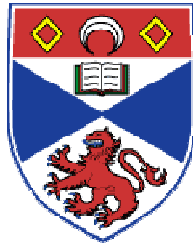# CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA08/05

The Effects of Government Spending Shocks on Consumption under Optimal Stabilization[*]

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THIS VERSION: JANUARY 2009

ABSTRACT

Economic theory has yet to come up with a general guidance regarding the dynamic effects and welfare implications of shocks to public spending. With the aim to provide a theoretical benchmark, we analyze if a rise in private consumption following an exogenous rise in government spending is a feature of the economy under optimal stabilization in a standard New Keynesian setting augmented for the presence of liquidity-constrained agents and non-separable preferences. Our results provide little evidence in support of a crowding-in effect under .timelessly optimal policy.

JEL Classification: E21, E52, E61, E63.
Keywords: Consumption, Government Spending, Optimal Monetary and Fiscal Policy, Non-Separable Preferences, Non-Ricardian Agents.

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1. Introduction

We model an economy in which both liquidity constraints and non-separable preferences can be introduced as a straightforward generalization of a standard New Keynesian model, and seek an answer to the question: Does a rise in private consumption follow an exogenous rise in government spending under optimal stabilization?

Analyzing the effects of changes in government spending on private consumption is important for understanding the effects of fiscal policy on people's welfare. Private consumption is the largest component of aggregate demand and is also assumed to be a principal determinant of agents' welfare. Economic theory has yet to come up with a general guidance regarding the dynamic effects and welfare implications of shocks to public spending. The insights offered so far usually depend on the nature of the simultaneous changes in policy variables. For example, according to simple textbook Keynesian models, private consumption should rise in response to a rise in government spending. The magnitude of the effect depends on the exact combination of tax and debt finance used to fund increased public spending. The effect on aggregate output also depends on changes in investment, the response in which crucially depends on monetary policy determining the interest rate prevailing in the economy. More formal analyses normally rely on well-established but still ad-hoc rules for the conduct of monetary and fiscal policy with the results again being dependent on the calibration of the simultaneous policy responses. The contribution of this paper is to investigate the nature and causes of the response in private consumption following an exogenous positive innovation to government spending under *optimal* conduct of monetary and fiscal policy through relevant instruments, with a view to provide a benchmark for future theoretical work in a New Keynesian environment.

Our primary focus is on private consumption, since most theoretical and empirical studies now suggest that a rise in government spending would increase aggregate output. There is much less of a consensus as regards the effects on private consumption. The Keynesian concept, in which agents finance consumption out of current income, has been summarized above. Most modern-day macroeconomic models follow the neoclassical tradition of assuming infinitely-lived consumption-smoothing agents. Such models would normally predict a fall in consumption due to a negative wealth effect generated by the need to raise taxes in the future to finance current fiscal expansion. The empirical literature provides equally conflicting answers.[1]

Recently, it has been investigated in both theoretical and empirical literature if departures from standard assumptions about consumer behaviour could provide more support for the Keynesian view. Galí et al. (2007) and Erceg et al. (2006) find support for the proposition that presence of 'hand-to-mouth' consumers in the economy can be associated with crowding in of private consumption. On the other hand, Rossi (2007) could not replicate the crowding-in result of Galí et al. (2007) in a standard New Keynesian setup with such rule-of-thumb behaviour when taxation is distortionary rather than lump sum. Coenen et al. (2007) only find a negligible positive response in consumption, driven by the wealth effect of an improvement in terms of trade in their open economy setup. The empirical literature is equally split on the question of the nature of the consumption response, whilst finding evidence in favour of non-standard descriptions of consumer behaviour.[2] The papers referred to here have assumed

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[1]See Galí et al. (2007) for a thorough review of both theory and evidence as well as some new results. Most recently, Ramey (2008) shows that the crowding-in result often found in VAR-based analyses, most notably Blanchard and Perotti (2002) and Galí et al. (2007), vanishes if an alternative identification method is used to isolate fiscal shocks. She raises the issue that some of the 'unanticipated' shocks identified in VAR models may actually have been anticipated.

[2]See Coenen and Straub (2005), López-Salido and Rabanal (2006) and Forni et al. (2007).

a simple-rule-based conduct by monetary and fiscal authorities. Our analysis also incorporates departures from the standard description of consumer behaviour as a straightforward generalization of a baseline case but looks at the effects of exogenous changes in government spending on private consumption from a normative perspective.[3]

We find that a rise in private consumption following a rise in government spending is generally not a feature of the economy under optimal stabilization even if the description of consumer behaviour departs from the conventions of macroeconomics. A crowding-in effect only emerges in circumstances that might be difficult to reconcile with reality in advanced economies. For instance, this is the case in an economy without liquidity constraints in which agents are highly risk-averse or in an economy with a large share of liquidity-constrained agents, very high labour supply elasticity and very low risk aversion.

Our framework is a standard New Keynesian economy in which prices are sticky and preferences can be made non-separable. We augment this framework for the presence of liquidity-constrained agents whom we shall henceforth refer to as *non-Ricardian*. We study dynamics under 'timelessly optimal' monetary and fiscal policy in this economy using a linear-quadratic setup.[4] In a conceptually related analysis, Bilbiie (2008) characterizes optimal discretionary and timelessly optimal monetary policy. Important simplifying assumptions that underlie his setup are that neither the fiscal consequences of monetary policy nor the first-order effects of stabilization policy are considered.[5] In our framework, where

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[3]The importance of exploring the normative angle was also suggested in the concluding remarks of Galí et al. (2007).

[4]Briefly, 'timelessly optimal' policies are policies the policy maker wished to have committed himself to had he made the decision about current policy in the distant past. See Woodford (2003), for instance, for a thorough explanation of the concept.

[5]Bilbiie (2008) also finds that the presence of liquidity-constrained agents beyond a threshold share may induce a change in the sign of the slope coefficient in the aggregate demand relationship with output rising in response to a rise in the real interest rate. There are associated

monetary and fiscal policy have to be coordinated to attain the optimal outcome and in which stabilization policy has level effects, these first-order effects turn out to play a key role in explaining optimal dynamics in the model.

The rest of the paper is organized as follows. Section 2 sets out the microeconomic foundations of the model. Section 3 presents the model of the linear economy and the quadratic objective function of the policy maker that follow from the micro-foundations. In Section 3 we also characterize the optimal dynamics of the economy using 'specific targeting rules' of Svensson (2002, 2003). It is a feature of our analysis worth emphasizing that the policy problem of the non-Ricardian economy as well as the optimal policy rules can be presented as a generalization of the baseline setup with Ricardian agents only, with the functional forms unaffected by the presence of non-Ricardian behaviour. The effects of the rise in government spending on private consumption and the determinants of the consumption response, including the influence of the presence of non-Ricardian agents, are discussed in Section 4. Section 5 looks at possible ways of operationalizing optimal policy and discusses equilibrium dynamics more broadly. Section 6 concludes.

2. The economy

In this section, we present a general equilibrium framework in which liquidity-constrained agents make up a stable proportion of all agents in the economy. We allow for heterogeneity among agents in terms of access to the asset market but our setup enables us to maintain much of the tractability of the representative agent framework.[6] This feature of the analysis then facilitates the use of modern

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determinacy issues, which have also been investigated in Rossi (2007) and Leith and von Thadden (2008). See section 5 for further discussion.

[6]This is partly due to the way preferences of individuals are described and partly due to the formulation of the government's objective. More discussion will follow.

methods of optimal policy determination.

2.1. Consumers

Consider an economy inhabited by a continuum of agents indexed by $k \in [0,1]$. The agents' utility is increasing in consumption C and leisure $(1 - H)$. As in Galí et al. (2004), we assume the following functional form for the utility of agents

$$u = \frac{[C\,(1 - H)^{\omega}]^{1 - \widetilde{\sigma}^{-1}}}{1 - \widetilde{\sigma}^{-1}} \tag{2.1}$$

with $\omega > 0$ and $\widetilde{\sigma}^{-1} \geqslant 0$. This function has the property that the implied marginal utility of consumption also explicitly depends on hours worked.[7] For $\widetilde{\sigma}^{-1} < 1$, $u_{CH} < 0$, meaning that a given extra unit of consumption raises welfare more if coupled with fewer hours worked. The opposite holds for $\widetilde{\sigma}^{-1} > 1$. When preferences are separable, $u_{CH} = 0$. This is the case in our baseline scenario with $\widetilde{\sigma}^{-1} = 1$.

Let us assume that the agents are identical in all aspects except for their access to the asset market. Agents indexed $k \in [0, \lambda]$ have no access to the asset market, whilst agents $k \in (\lambda, 1]$ can smooth consumption over time by varying their holdings of one-period nominal government debt—the only type of asset available in the economy.

2.1.1. Non-Ricardian agents

Agents who have no access to the asset market have to rely on current after-tax wage income to finance consumption. It can be shown that given a simple budget constraint that makes consumption equal to the after-tax wage, the period utility function of the form (2.1) is maximized if the liquidity-constrained agents supply

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[7]Marginal utility of consumption is given by $u_C = C^{-\widetilde{\sigma}^{-1}}\,(1 - H)^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)}$.

labour

$$H^{NR} = \frac{1}{1+\omega} \tag{2.2}$$

in which case the optimal consumption of these agents is given by

$$C_t^{NR} = \frac{1-\tau_t}{1+\omega} \frac{W_t}{P_t} \tag{2.3}$$

for all t. The variable τ denotes tax on wage income, W is the economy-wide nominal wage rate and P is the price index. Constant labour supply by non-Ricardian agents over time and across states of nature facilitates aggregation in the model.

2.1.2. Ricardian agents

The problem to be solved by the Ricardian agents—as we shall refer to the agents who smooth consumption over time—can be written as a problem of a representative agent choosing a sequence $\left\{C_T^R, H_T^R, b_T^R\right\}_{T=t}^{\infty}$ to maximize

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{\left[C_T^R \left(1-H_T^R\right)^{\omega}\right]^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}} \tag{2.4}$$

subject to

$$C_T^R + \frac{b_T^R}{1+i_T} = (1-\tau_T)\frac{W_T}{P_T}H_T^R + D_T + b_{T-1}^R \frac{P_{T-1}}{P_T} \tag{2.5}$$

for all $T \geqslant t$. The variable b^R stands for $(1+i)B^R/P$ in which B^R is the stock of nominal, one-period government debt held by the Ricardian agents. The nominal interest rate is denoted i. While the non-Ricardian agents are workers only, Ricardian agents hold stakes in firms. The variable D denotes dividends received on the basis of ownership of firms.

Combining the first-order conditions with respect to C^R and H^R from the above problem yields (with $T = t$)

$$C_t^R = \frac{(1 - \tau_t)}{\omega} \frac{W_t}{P_t} \left(1 - H_t^R\right) \tag{2.6}$$

and we also obtain the Euler equation

$$\beta \frac{E_t \left[\left(C_{t+1}^R\right)^{-\tilde{\sigma}^{-1}} \left(1 - H_{t+1}^R\right)^{\omega\left(1 - \tilde{\sigma}^{-1}\right)} \right]}{\left(C_t^R\right)^{-\tilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1 - \tilde{\sigma}^{-1}\right)}} \frac{(1 + i_t)}{(1 + E_t \pi_{t+1})} = 1, \tag{2.7}$$

in which $E_t \pi_{t+1}$ is expected inflation with $\pi_t = P_t/P_{t-1} - 1$. The relationship (2.7) solved in a multi-period form also defines the asset pricing kernel $Q_{t,T}$.

2.1.3. Aggregation

For aggregate consumption in our economy, it holds that

$$\begin{aligned}
C_t &= \int_0^\lambda C_t^{NR} dk + \int_\lambda^1 C_t^R dk \\
&= \lambda C_t^{NR} + (1 - \lambda) C_t^R. \tag{2.8}
\end{aligned}$$

A similar relationship holds for labour supply

$$H_t = \lambda H^{NR} + (1 - \lambda) H_t^R. \tag{2.9}$$

Since our asset holders are identical in all aspects, the holdings of assets will be distributed among them uniformly across time and state of nature. If aggregate asset holdings in the economy are denoted B, it follows that

$$B_t^R = \frac{B_t}{1 - \lambda}$$

for all t. In aggregate then, labour supply and consumption respectively are given by

$$H_t = \frac{\lambda}{1 + \omega} + (1 - \lambda) H_t^R, \tag{2.10}$$

$$C_t = \frac{(1 - \tau_t)}{\omega} \frac{W_t}{P_t} (1 - H_t). \tag{2.11}$$

Combining (2.2), (2.3), (2.6) and (2.11), yields

$$C_t^{NR} = \frac{\omega}{1 + \omega} \frac{C_t}{(1 - H_t)}, \tag{2.12}$$

$$C_t^{R} = \frac{C_t}{(1 - \lambda)} \left[1 - \frac{\omega \lambda}{(1 - H_t)(1 + \omega)} \right]. \tag{2.13}$$

We can thus express all variables in terms of aggregate variables and carry on solving the model using standard methods developed to identify optimal policy in representative agent frameworks.

2.2. Firms

Let us assume a continuum of monopolistically competitive producers of differentiated intermediate goods (indexed j). These goods then serve as an input in the production of a single final good. The production technology of the final good—produced by a representative firm operating in a perfectly competitive environment—is described by a Dixit-Stiglitz (1977) aggregator

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.14}$$

where $y(j)$ is the quantity of an intermediate good used in the production of Y. The coefficient ε denotes the constant elasticity of substitution between individual goods. A simple cost minimization exercise by final goods producers yields the expression for the demand for intermediate good j

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} \tag{2.15}$$

and the aggregate price index

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}. \tag{2.16}$$

Let us also assume that the production of the intermediate goods is described by the production function

$$y_t(j) = H_t(j)^{1/\alpha} \tag{2.17}$$

with $\alpha > 1$. In equilibrium it holds that

$$
\begin{aligned}
H_t &= \int_0^1 H_t(j)\, dj \\
&= Y_t^\alpha \delta_t
\end{aligned} \tag{2.18}
$$

with

$$\delta_t = \int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon\alpha} dj \tag{2.19}$$

denoting price dispersion.

The producers of intermediate goods maximize profits given by

$$\Upsilon(j) = p(j)\, y(j) - W H(j). \tag{2.20}$$

They do so in a forward-looking way, evaluating an expected stream of profits. We assume staggered price setting as put forward by Calvo (1983) with $\gamma \in (0,1)$ denoting the probability for a firm of charging unchanged prices in any period. With p_t^* being the price chosen for period t by all firms who can re-optimize their prices, the first order condition from this problem is written as

$$E_t \sum_{T=t}^{\infty} \gamma^{T-t} Q_{t,T} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\varepsilon} \left[(1 - \tau_T) - \mu \frac{W_T}{P_T} \alpha Y_T^{\alpha-1} \left(\frac{p_t^*}{P_T}\right)^{-\varepsilon(\alpha-1)-1}\right] = 0 \tag{2.21}$$

and the dynamics of the price level is then given by

$$P_t = \left[(1-\gamma)\, p_t^{*1-\varepsilon} + \gamma P_{t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{2.22}$$

2.3. Government

The government raises revenues T via *distortionary* taxes on wage income to finance exogenous government spending G. It issues one-period nominal bonds to

bridge the gap between taxation and spending. The government, therefore, faces the flow budget constraint

$$B_t = (1 + i_{t-1}) B_{t-1} - P_t s_t \qquad (2.23)$$

where B denotes the volume of one-period nominal bonds issued by the fiscal authority and $s = T - G$ is the primary budget surplus. This constraint can be rewritten as

$$\frac{b_t}{(1 + i_t)} = \frac{b_{t-1}}{(1 + \pi_t)} - s_t. \qquad (2.24)$$

We assume G follows an autoregressive process described by

$$\widehat{G}_t = \rho_G \widehat{G}_{t-1} + \varepsilon_{G,t},$$

in which $\varepsilon_{G,t}$ is an i.i.d. shock to government spending and $\rho_G \in [0,1)$.[8]

Monetary and fiscal authorities, the two branches of the central government, coordinate their actions to ensure that social welfare given by the discounted sum of weighted period utilities of Ricardian and non-Ricardian agents

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \lambda u_T^{NR} + (1 - \lambda) u_T^{R} \right\} \qquad (2.25)$$

is maximized. Arguably, maximizing the discounted value of weighted period utilities is a valid representation of social welfare if lack of access to the asset market comes from constraints rather than individual preferences.[9]

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[8]In general, $\widehat{x}$ will denote the percentage deviation of variable x from its steady state value $\overline{x}$. $\widehat{G}_t$ is an exception and is defined by $\left(G_t - \overline{G} \right) / \overline{Y}$ where $\overline{Y}$ denotes steady-state output.

[9]See Bilbiie (2008). Such a specification of the policy objective is also helpful, as it facilitates the derivation of the approximate Ramsey problem. An alternative way of setting up the same policy problem would be to assume that agents receive a signal whether they have or have no access to the asset market in the beginning of each period. Amato and Laubach (2003) have used this approach to introduce inertial rule-of-thumb behaviour into a framework similar to ours. Such a setup—arguably a less intuitive one in present circumstances—would necessitate some further assumptions to make sure the transversality condition is satisfied and that there is no need to track the distribution of assets as a separate state variable.

There are several ways to proceed from here. In this paper, we solve for the approximate optimal plan by formulating a linear-quadratic approximate policy problem. For models where stabilization policy has significant first-order welfare effects, which happens when there are non-zero linear terms in the second-order approximation to social welfare, the construction of a second-order-accurate welfare ranking criterion requires a second-order approximation to the structural equations. These are then used to substitute out the linear term from the approximation to social welfare. One thus obtains a welfare objective expressed purely in second-order terms with the first-order effects preserved in an implicit form. Without such a quadratic formulation of the approximate objective function, one would obtain inaccurate ranking of alternative policies examined in a purely linear system.[10] In the next section, we present the structural elements of the approximate problem. The derivation follows the steps in Benigno and Woodford (2003) and is not presented in this paper.

3. The macroeconomic model and the policy problem

The micro-foundations discussed in the previous section imply a simple New Keynesian model of the macroeconomy. The model we present here appears to be very similar to Benigno and Woodford (2003). We left the notation largely unchanged to indicate that we can present the economy with non-Ricardian agents as a generalization of the framework in which consumption smoothing applies to all consumers. The main difference here is that some key parameters of the model, such as the costliness of volatility in the target variables or the target level of output, will be a non-trivial function of the share of liquidity-constrained agents in the economy. Whilst we do not provide detailed derivations of the following equations here, a technical annex to the paper contains the derivations together

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[10]See Benigno and Woodford (2003, 2006) for an extensive treatment.

with the definitions of coefficients and variables resulting from the derivations. In the annex, we also provide indicative plots of calibrated values of the key parameters of the linearized model as a function of the share of non-Ricardian agents.[11] Further detailed discussion will follow in sections 4 and 5 of the paper.

The supply side of the economy is characterized by the following forward-looking New Keynesian Phillips curve

$$\pi_t = \kappa y_t + \chi_\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) + \beta E_t \pi_{t+1}. \tag{3.1}$$

The supply equation links current inflation π to the welfare-relevant output gap y, deviation in taxes and expected future inflation. The output gap here is defined as the difference between the actual deviation in output from its steady state and its 'target deviation' $\widehat{Y}^*$, where the latter follows from the approximation to welfare. The target deviation $\widehat{Y}^*$ is a function of the exogenous shock only and hence is independent of policy. Its magnitude also depends on structural parameter values. In general, it is different from the 'natural rate of output' commonly referred to in the literature on monetary policy as the level of output consistent with price stability. The 'efficient deviation' in the tax rate $\widehat{\tau}^*$ is the deviation that would offset the cost-push pressure resulting from the increase in government spending.[12]

The government's flow budget constraint can be shown to yield the following fiscal sustainability condition expressed in terms of the 'gap-variables' in (3.1)

$$
\begin{aligned}
\widehat{b}_{t-1} - \pi_t - \Phi^{-1} y_t + \varphi_t &= (1-\beta) \left[f_y y_t + f_\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) \right] \\
&\quad + \beta E_t \left[\widehat{b}_t - \pi_{t+1} - \Phi^{-1} y_{t+1} + \varphi_{t+1} \right].
\end{aligned} \tag{3.2}
$$

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[11]A working paper version of the paper with a detailed technical annex is available on request from the author or can be downloaded from http://www.st-andrews.ac.uk/cdma/m.horvath.html.

[12]The cost-push pressure arises when $\widehat{Y}^*$ does not correspond to the 'natural rate'. See Benigno and Woodford (2003).

This difference equation is in fact the linearized version of the familiar condition of fiscal sustainability that equates the value of outstanding liabilities to the value of future discounted primary budget surpluses. φ is the 'fiscal stress' term introduced in Benigno and Woodford (2003) as a composite measure of the consequences for fiscal solvency of the spending shock. The coefficient Φ^{-1} is the slope of the aggregate demand relationship linking output to real interest rate, while f_y and f_τ are the elasticities of primary surplus (as valued in terms of marginal utility of consumption) to output and the tax rate respectively.[13]

Finally, it follows from (2.25) that the central government conducts monetary and fiscal policy in a coordinated fashion to minimize the quadratic loss function

$$L_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \frac{1}{2} q_y y_T^2 + \frac{1}{2} q_\pi \pi_T^2 \right\}. \tag{3.3}$$

The coefficients q_y and q_π stand for the costliness of output gap and inflation volatility respectively. Following Benigno and Woodford (2003), the quadratic objective (3.3) is derived in a way to be able to provide a second-order accurate welfare ranking of alternate policies in the presence of non-negligible level effects, whilst the structural equations (3.1) and (3.2), together with appropriate initial commitments, are accurate only up to the first order. The commitments referred to are the policy maker's commitments regarding values of endogenous variables in period t the expectations of which would have been relevant for the determination of equilibrium in period $t - 1$. The specification of these commitments would follow from the long-run solution to the model. Policies that minimize (3.3) and satisfy the linear constraints (3.1) and (3.2) as well as these commitments are necessarily time-consistent. The policy maker has no incentive to deviate from

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[13]The solvency condition is 'priced' in units of marginal utility of consumption of Ricardian agents, following a substitution for the interest rate in (2.24) from the Euler equation (2.7). The aggregate demand relationship is in effect the first-order approximation to the Euler equation, hence the appearance of Φ^{-1} in the fiscal solvency constraint.

them over time. They are also optimal from a 'timeless perspective'.

The first-order conditions from this policy problem can be combined to obtain the 'specific targeting rules' in the sense of Svensson (2002, 2003)

$$E_t \pi_{t+1} = 0 \tag{3.4}$$

and

$$\pi_t + \frac{n_\phi}{m_\phi} \pi_{t-1} - \frac{\omega_\phi}{m_\phi} (y_t - y_{t-1}) = 0 \tag{3.5}$$

for all t. These rules define the relationships between aggregate variables that the monetary and fiscal branches of the central government authority should aim to bring about in a coordinated fashion. Again, we preserved the notation from Benigno and Woodford (2003). However, the coefficients in (3.5) will be a function of λ. The system comprising these targeting rules and the structural equations (3.1) and (3.2) defines the optimal dynamics of the economy. The following proposition holds.

Proposition 3.1. *Let optimal policy be characterized by (3.4) and (3.5). Commitment to policy conduct according to these rules ensures a determinate equilibrium in the economy given by (3.1) and (3.2) for all parameter values.*

Proof. We have stressed several times that our model and its solution is isomorphic in terms of functional forms to Benigno and Woodford (2003). Following the steps in Appendix A.11 in Benigno and Woodford (2003), it is straightforward to show that the system of structural equations and the first-order conditions from the policy problem can be reduced to a dynamic system of the form

$$E_t \mathbf{z}_{t+1} = \mathbf{M}\mathbf{z}_t + \mathbf{N}\xi_t,$$

in which $\mathbf{z}$ is a vector of endogenous variables and ξ is an exogenous disturbance. $\mathbf{M}$ and $\mathbf{N}$ are matrices of coefficients. $\mathbf{M}$ can be shown to be lower triangular,

with diagonal elements (eigenvalues) that are independent of λ, and satisfying the Blanchard and Kahn (1980) determinacy conditions. ∎

4. Effects of government spending on private consumption

We study numerical calibrations of the optimal dynamics derived in the previous section. The baseline case is the optimal economy without non-Ricardian agents whose preferences are separable (logarithmic). We then examine if departures from conventional modelling of consumer behaviour—as suggested by Campbell and Mankiw (1989), Mankiw (2000) and Basu and Kimball (2002)—could significantly alter the conclusions regarding the effect of government spending on consumption under optimal policy.[14] A link between the way consumer behaviour is modelled and the nature of the response in consumption to a spending shock has been suggested in the context of models assuming a simple rule-based conduct of policy, as explained in the introduction.

4.1. Calibration

We calibrate the model of the optimal economy using the following structural parameter values. The *quarterly* discount rate, β, is calibrated to a commonly used value of 0.99, implying an annualized steady-state rate of interest just over 4 percent. The consumption share of national income, c, is 0.8. The value of $\widetilde{\sigma}^{-1}$ is set to 1 in the baseline calibration, which implies a log-linear (separable) functional form and is varied from low values of around 0.13 estimated in Rotemberg and Woodford (1997) to high values exceeding 1 commonly found in the literature which estimates the elasticity of consumption to the real interest rate to be very low. The value of ω in the utility function is calibrated so that the Frisch elasticity

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[14]In section C of the technical annex, we present a simple algebraic analysis that helps linking our model to earlier literature on consumer behaviour.

of labour supply (given by $\left(1 - \overline{H}\right)/\overline{H}$ from (2.11)) takes on a value of 1, as in Galí et. al (2004). Apart from this baseline case, we consider a significantly less elastic labour supply function and a significantly more elastic one. We assume an approximate 11 percent price markup in the product market, arising due to imperfect competition among intermediate goods producers. We set $\alpha = 1.25$ so that the production function governing the production of intermediate goods is of decreasing-returns-to-scale type. The price stickiness parameter in the Calvo-pricing model γ has been set to 0.65. The steady state labour income tax rate is 30 percent. These parameter values imply a steady-state surplus-to-GDP ratio of 0.016 and hence $\bar{b}/\overline{Y} = 1.6$ or a debt level of 40 percent of steady-state output on an annual basis.

Solving the model for the optimal steady state yields two solutions for the steady-state output, one of which represents a special case with the Ricardian agents consuming no leisure $(\overline{H}^R = 1)$ so that $\overline{H} = \frac{1+\omega-\omega\lambda}{1+\omega}$. By (2.6), this implies a corner solution case, a case of zero consumption for Ricardians in the steady state. A positive deviation from this steady state then implies an infinite increase in utility for Ricardian agents and for the whole economy too. We therefore concentrate on the interior solution. The corresponding steady-state level of output is independent of λ. This follows from the fact that the economy-wide real wage rate, and hence also marginal cost, depend only on aggregate variables, as defined in (2.11). The share of non-Ricardian agents in the economy λ is varied from 0 to an upper bound of lambda $\overline{\lambda}$. This upper bound represents the share of non-Ricardian agents at which Ricardian agents stop supplying labour to the economy. This result arises, as it holds in the steady state that $\overline{H}^R \leqslant \overline{H} < \overline{H}^{NR}$ for all λ.

In the analysis presented here, including all calibrations in the sensitivity analysis, the parameter values yield positive coefficients q_y and q_π in the loss

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function (3.3). The objective function is then convex and the optimal solutions presented in the next section are consistent with minimum losses in terms of the loss function (3.3).

4.2. Optimal consumption dynamics

We now turn to examining optimal consumption dynamics following an exogenous, serially correlated rise in government spending of 1 percent of steady-state output with a persistence parameter $\rho_G = 0.9$.[15] The impulse response function for private consumption under different calibrations of agents' preferences and price stickiness in an economy without liquidity-constrained agents are plotted in the top panel of Figure 4.1.[16] The bottom panel contains the same for aggregate output. Two properties need to be highlighted here. First, the optimal behaviour of private consumption and output is non-stationary in an environment with nominal rigidity. This is in line with the observations made in Benigno and Woodford (2003) and Schmitt-Grohé and Uribe (2004). Under flexible prices though, inflation can be freely used to deal with the fiscal consequences of the government spending shock and taxes vary only to ensure the output gap is zero throughout. Hence, once the shock dies out and the steady-state level of output becomes the target level, all variables return to their pre-shock steady state levels. Second, we see that the optimal initial response in private consumption is consistently negative for most calibrations. However, it is also clear that higher degrees of risk aversion and higher values of labour supply elasticity tend to make the response less negative. In fact, when risk aversion is very high, the optimal response ultimately becomes positive. On the other hand, our analysis finds the initial response in output to be consistently positive for all reported calibrations.

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[15]Authors normally estimate or calibrate this parameter to attain very high values. The literature cited in this paper has found or used values ranging from around 0.8 to 0.975.

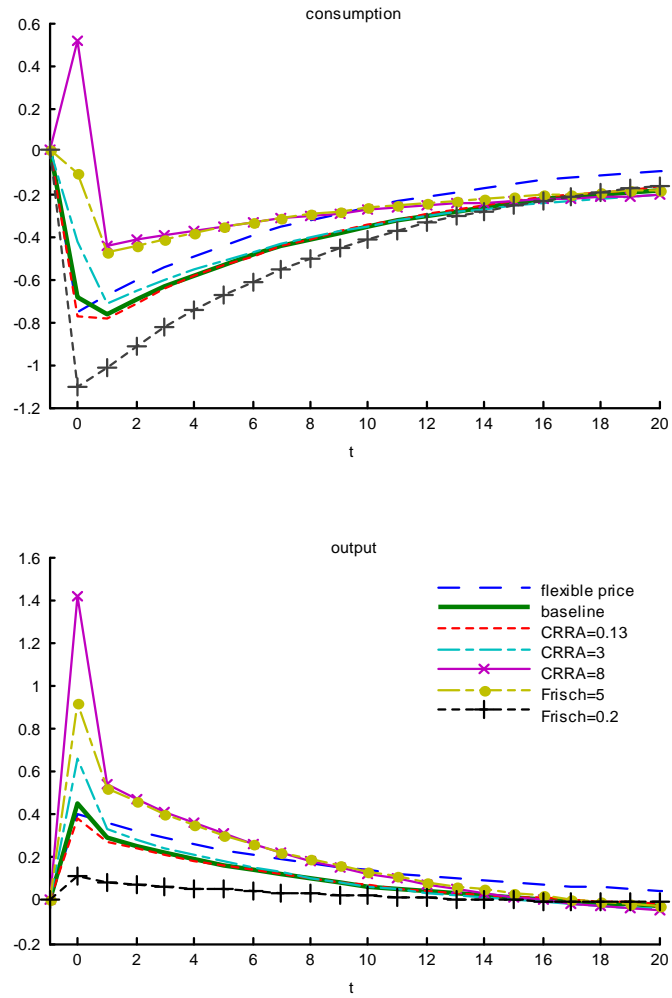[16]1 on the vertical axis denotes a one-percent deviation from the pre-shock steady-state value.

Figure 4.1: Private consumption and output dynamics (deviations from steady state) following a rise in government spending in an economy with Ricardian agents only

Next, we examine how the optimal initial response in private consumption changes as we introduce and gradually raise the share of non-Ricardian agents in the economy. Figure 4.2 plots the maximum response in private consumption and output for alternative calibrations of structural parameters as a function of the share of non-Ricardian agents.[17] We find that the above mentioned positive optimal response in consumption at high levels of risk aversion is generally not a feature of economies with non-Ricardian agents. However, a positive response in consumption can be shown to be consistent with optimal policy when the share of non-Ricardian agents is fairly high, labour supply elasticity is very high and the coefficient of relative risk aversion is low.

Neither of the situations when a positive private consumption response occurs is, however, likely to be easily reconcilable with reality in advanced economies. Hence, in our setup, crowding out of private consumption by government spending is generally consistent with timelessly optimal policy. Interestingly, we find that even output may in some cases fall as a consequence of increased government spending. We see this happening in economies with a moderate share of non-Ricardian agents when agents are highly risk averse or in economies with a high share of non-Ricardian agents, in which agents are moderately highly risk averse and labour supply is highly elastic.

4.3. The role of the first-order effects of stabilization policy

Whilst there are several factors playing a role in explaining optimal aggregate dynamics in the economy, it turns out that the first-order effects (or level effects) of stabilization policy play a dominant role among these.

In the context of the model, the level effects are a manifestation of a trade-off

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[17]Note that the mark need not correspond to the peak of the impulse response function. If the variable converges to a new steady state value which is greater than the level on impact, the new steady-state value is shown.
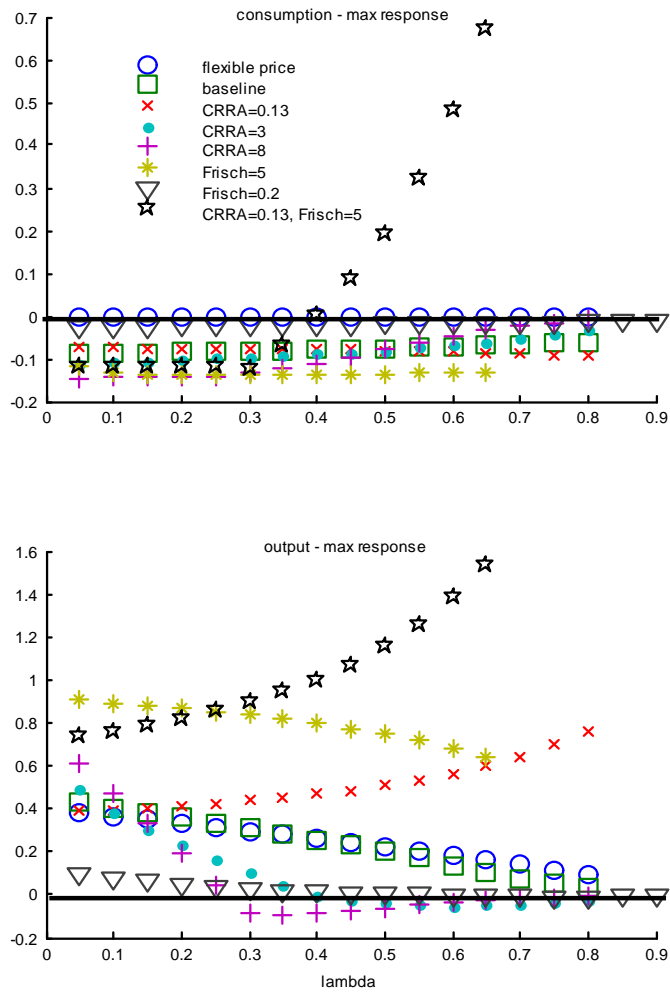
Figure 4.2: Maximum response in private consumption and output to a rise in government spending

between volatility and average outcomes. Forward-looking optimizing firms, when faced with a more stable environment, would either change prices less in response to a shock (ceteris paribus), leading to less price dispersion and higher efficiency, or, in other words, could allow for somewhat higher average levels of production and hence average marginal cost over time for a given chosen price.[18] Of course, the public finance implications of such a trade-off have to be taken into account too when assessing the first-order effects of a reduction in volatility. Having lower volatility necessitates higher average tax levels (and hence lower output levels) over time to compensate for the positive effect of volatility on tax receipts due to the convexity of the tax schedule.

As mentioned before, such first-order effects are implicit in the objective function (3.3) via quadratic terms substituted from the second-order approximations to the structural equations. The costliness of a given degree of output volatility, q_y, depends (positively) to a decisive extent on the welfare effects of the potential first-order output gains arising from lower volatility.[19] The measure of the costliness of volatility in turn determines the target level of output—that is the level around which we aim to stabilize the economy—as well as the optimal size of the output gap. The coefficient q_y is inversely related to both of them, as also seen in Figure B.1 in the technical annex. For a sufficiently

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[18]See the technical annex for details on the approximation. The story put forward here is related to Siu (2004) who explains that in a highly volatile environment, risk-averse producers would always set prices as if they expected a large, positive (inflationary) spending shock. This happens because they try to avoid a situation in which they would set prices too low and facing high demand, they would run losses. By contrast, if they set prices too high, the worst outcome is that the face zero demand and make zero profit.

[19]An increased average level of output could bring more utility through a rise in consumption but one also needs to account for the loss of utility due to the extra labour supply that goes with increased output levels. This is captured by the coefficient Θ_Y, which appears in the first two terms of the definition of q_y (see the technical annex). The first term in q_y then stands for the first-order welfare effects of a reduction in output and inflation volatility affecting firms' behaviour, while the second term brings into consideration the fiscal implications of reduced volatility. As explained intuitively above, they have the opposite sign.

low q_y, the optimal response in output becomes large enough to be consistent with a rise in consumption.

Given the preferences of agents, when risk aversion rises beyond the degree corresponding to logarithmic preferences, the net utility gain from a percentage increase in the average level of output shrinks and eventually even becomes negative. Hence, q_y falls, and volatility becomes less costly in welfare terms. Thus, with rising $\widetilde{\sigma}^{-1}$, we observe higher volatility, larger initial responses in output and hence also consumption. Ultimately, the implied consumption response becomes positive.[20]

It is, however, enough to have a very small degree of non-Ricardian behaviour present in the model for this crowding-in effect to disappear. It happens because the welfare gains from higher levels of output rise significantly, as we include non-Ricardian consumers enjoying extra consumption, whilst the marginal welfare cost of increased output actually falls due to the convexity of the agents' utility function in labour.[21] As a consequence, more stability becomes desirable and the magnitude of the initial response in output falls.

On the other hand, when labour supply elasticity is high and the degree of risk aversion is low, the welfare gains from extra consumption are relatively small, and also only small wage hikes are sufficient to induce the needed supply of extra labour by the Ricardians. There is thus little potential welfare gain to be reaped by the non-Ricardian agents from higher output levels. Moreover, the Ricardians' welfare function is then concave in labour causing that supplying extra units of labour becomes more costly in welfare terms as their share in the population falls. Then, as we increase the share of non-Ricardian agents, the welfare gains from

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[20]Recall that given the non-separability in preferences, there is complementarity between consumption and leisure (hours worked) and $\widetilde{\sigma}^{-1}$ also affects the disutility of labour. See section 2.1.

[21]Recall that the steady-state labour supply of Ricardian consumers falls as we include hand-to-mouth consumers.

more stability are decreasing. More volatility will become desirable and the initial response in consumption can again become positive.

In all other circumstances, following a similar rationale, the desirable degree of volatility is not large enough to be consistent with a rise in private consumption at times when the spending shock affects the economy. Or, looking at it from reverse angle, a positive response in private consumption would require an excessive degree of volatility in the economy, which would hurt agents through lower average welfare over time.

5. Optimal operational policy and equilibrium dynamics

It is a distinctive feature of our paper that policy responses to the exogenous shock to government spending are optimal. It is therefore of notable interest to characterize optimal policy action at operational level, to understand the motivation behind it and to explain how it affects the functioning of the economy in a broader sense. This is what we do in this section. First, we discuss optimal reaction functions for policy instruments as well as some determinacy issues that have been raised in the literature by e.g. Bilbiie (2008), Leith and von Thadden (2008) and Rossi (2007). We will argue that whilst determinacy issues are not seriously affecting our analysis of operational policy, it is still preferable to define policy in terms of commitment to specific targeting rules, as specified in section 3. Then we provide a more detailed discussion of equilibrium dynamics, highlighting the ways presence of non-Ricardian agents can affect optimal dynamics.

5.1. Operational policy and determinacy

Combining the aggregate supply relationship (3.1) and the targeting rule (3.4), we obtain the operational optimal reaction function for the tax rate

$$\widehat{\tau}_t = \widehat{\tau}_t^* - \frac{\kappa}{\chi_\tau} y_t + \frac{1}{\chi_\tau} \pi_t. \tag{5.1}$$

To characterize optimal monetary policy via a reaction function for the interest rate, we need to introduce formally the demand side of our economy. The log-linearized version of (2.7) and the approximation to consumption imply the following intertemporal 'IS' relationship expressed in terms of welfare-relevant output gaps

$$y_t = E_t y_{t+1} - \Phi \left(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^* \right). \tag{5.2}$$

The variable $\widehat{r}^*$ here represents the deviation in the interest rate that is consistent with the preference-driven target deviation in output $\widehat{Y}^*$ under stable prices. $\widehat{r}^*$ depends on exogenous real variables only and hence, cannot be affected by government policy. $\widehat{i}_t = \log \frac{1+i_t}{1+\overline{i}}$, where $\overline{i}$ is the steady state interest rate determined by the rate of time preference. Combining this equation with (3.5) in an appropriate manner gives us the 'expectations-based' reaction function for the interest rate[22]

$$\begin{aligned}
\widehat{i}_t &= \widehat{r}_t^* + E_t \pi_{t+1} + \Phi^{-1} E_t y_{t+1} - \Phi^{-1} \frac{m_\phi}{\omega_\phi} \pi_t \\
&\quad - \Phi^{-1} \frac{n_\phi}{\omega_\phi} \pi_{t-1} - \Phi^{-1} y_{t-1}.
\end{aligned} \tag{5.3}$$

From this, it follows that the long-run response to inflation is given by

$$1 - \frac{\Phi^{-1}}{\omega_\phi} \left(m_\phi + n_\phi \right). \tag{5.4}$$

Generally speaking, the optimal dynamic solution presented in the previous section can be obtained as a unique and stable solution to the system of

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[22]See Evans and Honkapohja (2006). Expectations in this function are expectations of private agents as observed by the authorities. The reason for using this as opposed to many possible alternative specifications of the interest rate reaction function is that other specifications may lead to serious problems with the determinacy of the solution. Indeterminacy in relation to this specification of the reaction function is limited to a rather special case. See Proposition 5.1 below.

structural equations if policy is set according to optimal expectations-based reaction functions for policy instruments. There are special cases when such a solution cannot be obtained. This happens when the aggregate demand relationship becomes perfectly inelastic. This leads us to the following proposition:

Proposition 5.1. *The equilibrium solution implied by the system comprising equations (3.1), (3.2), (5.1), (5.2) and (5.3) is determinate if and only if $\Phi^{-1} \neq 0$.*

Proof. First, suppose $\Phi^{-1} = 0$. Dividing (5.2) by Φ, together with using $\Phi^{-1} = 0$ in (5.3), yields that any path of inflation and output gap solves both equations. We are thus left with only three equations with four unknowns: (3.1), (3.2) and (3.4). We obtain the latter by combining (3.1) with the reaction function for the tax rate. It is easy to show that such a system has multiple solutions. Solving (3.1), (3.2) in $t + 1$, using (3.4), and substituting from (3.1) into an integrated version of (3.2) for the 'tax gap' yields

$$\widehat{b}_t + \varphi_{t+1} = (1 - \beta) \, E_{t+1} \sum_{T=t+1}^{\infty} \beta^{T-t-1} \left(f_y - \frac{f_\tau \kappa}{\chi_\tau} \right) y_T.$$

This equation defines the state in period t. In the absence of a rule defining future output gap dynamic, there are multiple possible debt levels, which represent equilibria to which the system can jump following the shock.

Second, suppose $\Phi^{-1} \neq 0$. Then (3.1) and (5.1) imply (3.4), while we recover (3.5) when combining (5.2) and (5.3). We are thus left with the system which, according to Proposition 3.1, has a determinate solution for all parameter values. ∎

The fact that the IS relationship can swivel is an issue identified in Bilbiie (2008). Bilbiie referred to the phenomenon of having an upward-sloping aggregate demand relationship as 'inverted aggregate demand logic'. The reason for the phenomenon was explained as follows. A rise in the real interest rate initially acts

to induce a fall in consumption of optimizing agents. The implied reduction in aggregate demand depresses wages, which leads to increased profits distributed as dividends to optimizers. These profits may, if the share of rule-of-thumb agents is sufficiently large, lead to a positive wealth effect strong enough to more than offset the direct effect on optimizers' consumption. The analysis of determinacy in economies with non-Ricardian agents has then been taken forward to include rule-based fiscal policy in Rossi (2007) and Leith and von Thadden (2008). Leith and von Thadden found no evidence of a bifurcation analogous to Bilbiie's in their model unless capital was excluded from the model.

The aggregate demand relationship can swivel in our model too and in section D of the technical annex, we recover the story put forward by Bilbiie (2008). There is, however, a further element present in our analysis, which follows from non-separability of preferences and the dependence of marginal utility of consumption on hours worked, as explained in section 2.1. If non-separable preferences are assumed, marginal utility of consumption may be falling in consumption but need not in output, depending on a sufficiently large positive cross-derivative u_{CH}. Recall that u_{CH} is positive for $\widetilde{\sigma}^{-1} > 1$ and its size for Ricardian agents depends positively on the share of non-Ricardian agents. It follows that a rise in the real interest rate might induce a re-allocation of production in favour of today, as Ricardian agents adjust their choices of consumption and leisure simply to satisfy the Euler equation. These two explanations sometimes complement and sometimes counteract each other, depending on the calibration of $\widetilde{\sigma}^{-1}$ and λ. Overall, our analysis indicates that we are looking for a combination of $\widetilde{\sigma}^{-1}$ and λ to determine when the 'IS' relationship swivels. Figure 5.1 plots the combinations of $\widetilde{\sigma}^{-1}$ and λ leading to vertical aggregate demand relationships and hence indeterminacy in our instrument-rule-based model. Points above the curve correspond to economies with an inverted aggregate demand schedule.

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It follows from Proposition 5.1 and expression (5.4) that the bifurcation also implies a change in the optimal long-run response to inflation in the 'expectations-based' interest rate reaction function, which is greater than one for standard downward-sloping aggregate demand relationships and becomes less than one when consumption rises in response to a rise in the real interest rate.

To conclude this part, since there are no determinacy issues associated with commitment to specific targeting rules, we consider this a superior and preferable way of characterizing optimal policy. There are also other arguments not related to determinacy, spelled out most eloquently in Svensson (2003), why one might wish to characterize policy at a higher level of generalization.[23] Nevertheless, the dynamics of policy variables defined by the above reaction functions form an integral part of the explanation of equilibrium dynamics of endogenous variables.

5.2. Optimal dynamic adjustment

We discuss the dynamic adjustment in the optimal economy when $\widetilde{\sigma}^{-1} = 0.13$ and the Frisch labour supply elasticity equals 5. As explained in the previous section, higher population shares of non-Ricardian agents are associated with a crowding-in effect under optimal stabilization in such an economy. We have chosen this calibration as it illustrates the implications of the presence of non-Ricardian agents perhaps most strikingly. At the same time, there is no loss of generality, since the dynamics associated with no rise in consumption following the spending shock are in principle similar under different calibrations. Figure 5.2 plots the

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[23]In particular, we pick up one important related to the size of the coefficients in optimal policy rules, which is highly relevant for our reaction function for the tax rate. Svensson (2003) writes that (optimal) instrument rules with large coefficients could cause that 'the slightest mistake in calculating the argument of the reaction function would have grave consequences and result in extreme instrument-rate volatility'.
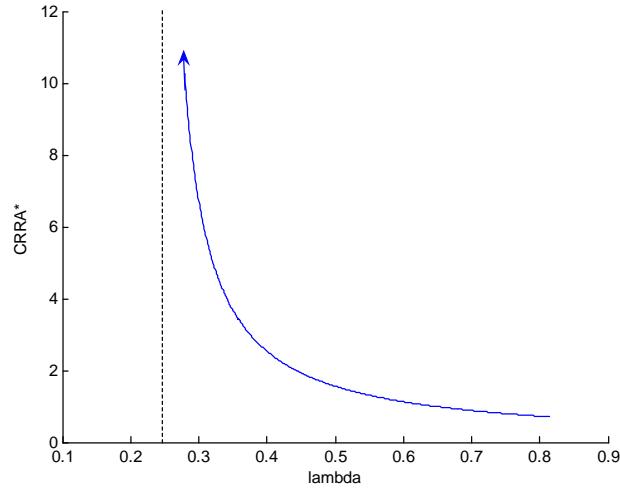
Figure 5.1: Parametric combinations leading to indeterminacy

impulse response functions.[24]

Our starting point is the behaviour of taxes. In models such as ours, the tax rate plays a role in ensuring price stability. The tax rate adjusts primarily to remove cost-push (or cost-pull) pressures by bringing the natural rate consistent with price stability closer to the welfare-driven target deviation $\widehat{Y}^*$.[25] When q_y is large and the target output response is small, which normally corresponds to the absence of a crowding-in effect, taxes need to rise to induce a reduction in the natural rate of output, which depends on tax policy, to offset cost-pull pressures. By contrast, when the welfare-efficient level of output, around which we wish to stabilize the economy, is large, taxes need to fall to offset cost-push pressures. In a slightly simplified language, if marginal cost would have to increase

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[24] 1 on the vertical axes again denotes 1 percent. Recall also that the model and hence also the inflation and interest rate responses correspond to quarterly values.

[25] Full stabilization is generally not possible when prices are sticky. There is thus also a usually relatively small output gap response. See Benigno and Woodford (2003).

'too much' to bring about a desired level of output, exerting an undue upwards pressure on prices, tax policy helps to ease the pressure on firms by lowering distortionary taxes on labour, tilting agents incentives towards labour as opposed to leisure. We can see this from the shape of the net real wage response, which also corresponds to the response in the consumption of non-Ricardian agents. The contemporaneous response in net real wages is in such a case large enough to offset the negative wealth effect on private consumption caused by the need to keep taxes permanently higher in the long term to finance a permanently higher debt level. Public debt is non-stationary, as is now commonly found in the literature with imperfectly flexible nominal adjustment.[26] This is an extreme version of tax smoothing, when taxes are held permanently higher (and output and consumption permanently lower) to avoid a more abrupt initial adjustment in the tax rate.

The interest rate policy must ensure the demand side of the economy adjusts to the desired target level. In line with textbook Keynesian models, the primary effect of a positive innovation to government spending is to push interest rates upwards. However, in order to bring the economy to its target position, if that position is sufficiently distant from the initial steady state, might require a cut in the interest rate. This is captured by $\widehat{r}^*$ in (5.3). Since the target level of output is much larger when one observes crowding in of consumption, the necessary cut is deeper in that case. Sometimes, when the target level of output is smaller, the net effect on the interest rate of the spending shock may be positive as shown in Figure B.1 in the technical annex.

A negative interest rate response, as displayed below, also relieves fiscal stress, by easing debt service burden, though in the case shown below, it is more than offset by the effects of the efficient tax dynamic. This is another reason why

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[26]See Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2004) and Chugh (2006), for instance.

we observe a somewhat larger response in the output gap (determining the wage bill), and also a somewhat larger equilibrium rise in public debt when λ is set at 0.5. Public debt covers the part of budgetary consequences of spending shock that is not inflated away or not covered by extra tax revenues (if any). In line with a general assertion from the literature, the inflation response is very small under price stickiness and stabilizes immediately following the initial impact of the shock, as the policy makers act to satisfy the target (3.4).

6. Concluding remarks

We have presented a normative analysis of the question of whether increases in government spending crowd in private consumption. We have done this in a standard New Keynesian framework which included the possibility of limited asset market participation by agents and non-separable individual preferences—two features that were suggested in positive work as potentially relevant in justifying the crowding-in effect predicted by Keynesian logic and found in some empirical work. Our results provide little support for a crowding-in effect under optimal stabilization policy. An increase in private consumption following a positive innovation in government spending would require an undue degree of volatility in the economy, which would hurt agents through lower average welfare over time. Hence, it is generally not a feature of the optimal economy.

Whilst our analysis sends out a fairly unambiguous message, let us point to a few issues that have not been dealt with in this paper and could affect its conclusions in either direction. We have used a framework and a solution method that represent the current state-of-the-art in macroeconomics, nevertheless, we have made two sacrifices in the name of tractability and policy-relevance. First, the absence of capital in the model seems to be the most obvious simplifying assumption. Second, in our model, wages adjust instantaneously to make sure the
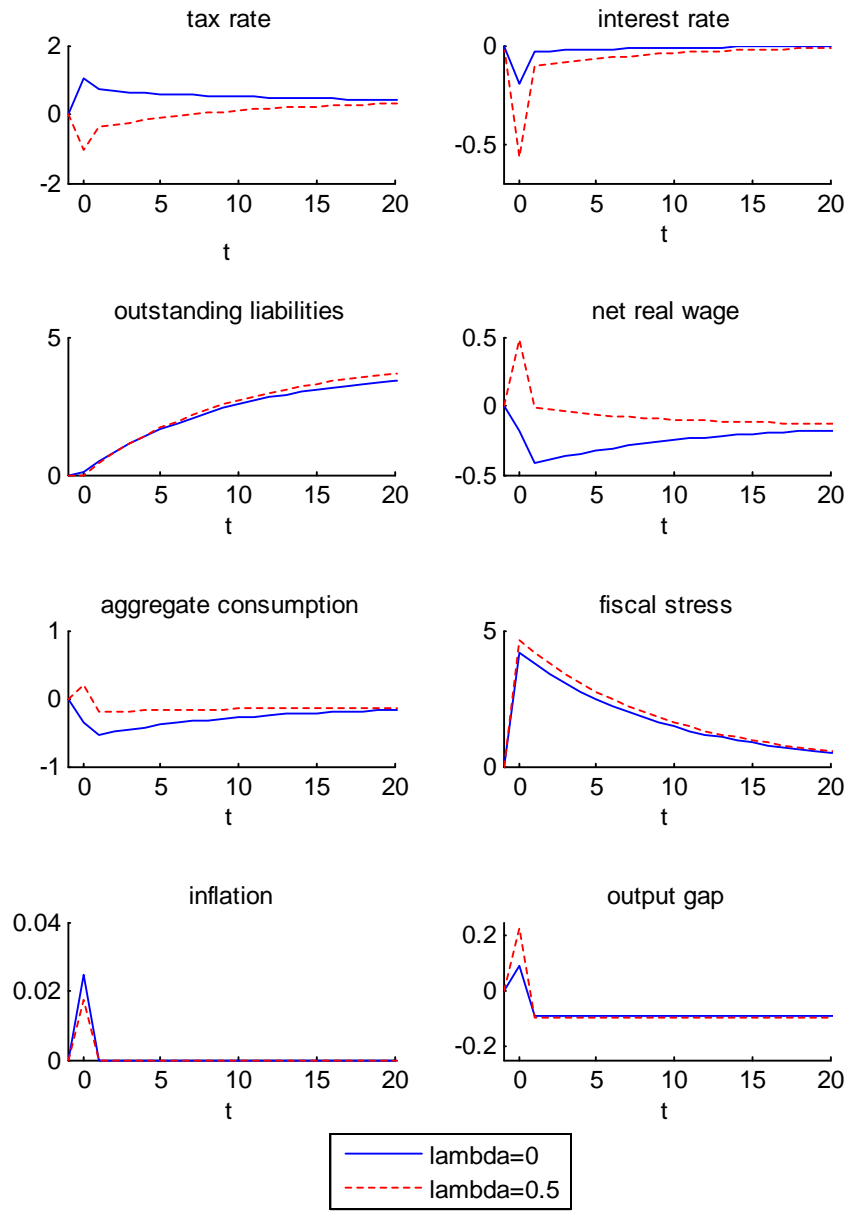
Figure 5.2: Equilibrium dynamics of endogenous variables

labour market clears. As argued in Christiano et al. (1997), this normally implies a sharp response in real wages which is not supported by empirical evidence. A different approach to modelling individual preferences and the labour market, as in Galí et al. (2007) for instance, could allow for nominal wage rigidities to be modelled alongside imperfectly flexible price adjustment in an economy where some agents are liquidity constrained.

This discussion suggests that analyzing optimal consumption dynamics in the context of a medium-scale macroeconomic framework such as Christiano et al. (2005) extended for the features of consumer behaviour used in this paper represents a potentially fruitful research agenda. Schmitt-Grohé and Uribe (2005) offer a method for solving such optimal policy problems numerically, though with significant sacrifices in terms of the tractability of the solution.

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The Effects of Government Spending Shocks on Consumption under Optimal Stabilization
Technical Annex

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January 2009

Abstract

This file complements the main text of the paper published under the title above. It provides details of the derivation of the structural equations and the policy objective, coefficient definitions, their numerical calibrations, more insight into the links between the analysis in the paper and the existing literature on consumer behaviour and an explanation of the potential causes of an inverted aggregate demand relationship in the model economy.

JEL Classification: E21, E52, E61, E63.

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A. The structural equations and the policy objective

In the first section of the annex, we provide the derivations that underlie the structural equations in section 3 of the paper. This includes definitions of the coefficients used in section 3 expressed in terms of the structural parameters introduced in section 2 of the paper.

The aggregate supply relationship

The aggregate supply relationship can be derived from the firm's optimization problem as follows. We assume Calvo pricing with γ being the probability of leaving prices unchanged in a given period. The firm is choosing the optimal price and the intertemporal first-order condition is written as

$$
E_t \sum_{T=t}^{\infty} \gamma^{T-t} Q_{t,T} Y_T \left(\frac{p_t(j)}{P_T} \right)^{-\varepsilon} \left[(1-\tau_T) - \mu \frac{W_T}{P_T} \alpha Y_T^{\alpha-1} \left(\frac{p_t(j)}{P_T} \right)^{-\varepsilon(\alpha-1)-1} \right] = 0.
$$
(A.1)

$Q_{t,T}$ is the stochastic discount factor which can be derived from the Euler equation. We can define $\omega_p = \alpha - 1$ and μ will replace $\mu = \varepsilon/(\varepsilon - 1)$ as the price markup due to imperfect competition in the intermediate goods market. After substituting from (2.6) for the real wage rate and using $p_t(i) = p_t^*$, we obtain a closed-form solution

$$
\frac{p_t^*}{P_T} = \left(\frac{K_t}{F_t} \right)^{\frac{1}{1+\varepsilon \omega_p}}
$$
(A.2)

with

$$
K_t = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \left(C_T^R \right)^{\left(1-\tilde{\sigma}^{-1} \right)} \left(1 - H_T^R \right)^{\omega\left(1-\tilde{\sigma}^{-1} \right)-1} \frac{\mu\alpha\omega Y_T^{\alpha}}{(1-\tau_T)} \left(\frac{P_T}{P_t} \right)^{\varepsilon(1+\omega_p)},
$$
(A.3)

$$
F_t = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \left(C_T^R \right)^{-\tilde{\sigma}^{-1}} \left(1 - H_T^R \right)^{\omega\left(1-\tilde{\sigma}^{-1} \right)} Y_T \left(\frac{P_T}{P_t} \right)^{\varepsilon-1}.
$$
(A.4)

The price index evolves according to

$$
P_t = \left[(1-\gamma) p_t^{*1-\varepsilon} + \gamma P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}
$$
(A.5)

which, together with (A.2), implies the following implicit definition of the inflation rate $\Pi_t = P_t/P_{t-1}$:

$$\left[\frac{1 - \gamma\Pi_t^{\varepsilon-1}}{1 - \gamma}\right]^{\frac{1+\varepsilon\omega_p}{\varepsilon-1}} = \frac{F_t}{K_t}. \tag{A.6}$$

The law of motion for price dispersion as defined in (2.18) is

$$\delta_t = \gamma\Pi_t^{\varepsilon(1+\omega_p)}\delta_{t-1} + (1 - \gamma)\left[\frac{1 - \gamma\Pi_t^{\varepsilon-1}}{1 - \gamma}\right]^{-\frac{\varepsilon(1+\omega_p)}{1-\varepsilon}}. \tag{A.7}$$

Let us further define

$$D_t = \left[\frac{1 - \gamma\Pi_t^{\varepsilon-1}}{1 - \gamma}\right]^{-\frac{1+\varepsilon\omega_p}{\varepsilon-1}} \tag{A.8}$$

$$= \frac{K_t}{F_t}. \tag{A.9}$$

As derived in Benigno and Woodford (2004), a second-order approximation to D_t as given by (A.9) can be written as

$$
\begin{aligned}
\frac{\widehat{D}_t}{1 - \beta\gamma} + \frac{1}{2}Z_t\widehat{D}_t &= z_t + \frac{\gamma}{1 - \gamma\beta}E_t\left[\widehat{D}_{t+1} + (1 + \varepsilon\omega_p)\pi_{t+1}\right] + \frac{1}{2}z_tX_t \\
&\quad + \frac{1}{2}\gamma\beta E_tZ_{t+1}\left[\widehat{D}_{t+1} + (1 + \varepsilon\omega_p)\pi_{t+1}\right] \\
&\quad - \frac{1}{2}\frac{\gamma}{1 - \gamma\beta}(1 - 2\varepsilon - \varepsilon\omega_p)E_t\pi_{t+1}\left[\widehat{D}_{t+1} + (1 + \varepsilon\omega_p)\pi_{t+1}\right] \\
&\quad + O(3).
\end{aligned}
\tag{A.10}
$$

Z_t, z_t and X_t are expressions and will be defined later. $E_t\pi_{t+1}$ refers to expected inflation with $\pi_t = P_t/P_{t-1} - 1$. A second-order expansion of (A.8) can in turn be written as

$$\frac{\widehat{D}_t}{1 + \varepsilon\omega_p} = \frac{\gamma}{1 - \gamma}\pi_t - \frac{1}{2}\frac{\gamma(1 - \varepsilon)}{(1 - \gamma)^2}\pi_t^2 + O(3). \tag{A.11}$$

Substituting for $\widehat{D}_t$ in (A.10) leads us to

$$V_t = E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\zeta z_T + \frac{1}{2}\zeta z_TX_T + \frac{1}{2}\varepsilon(1 + \omega_p)\pi_T^2\right\} + O(3) \tag{A.12}$$

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where $\zeta = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma(1+\varepsilon\omega_p)}$ and V_t stands for

$$V_t = \pi_t - \frac{1}{2}\frac{(1-\varepsilon)}{(1-\gamma)}\pi_t^2 + \frac{1}{2}(1-\beta\gamma)\pi_t Z_t + \frac{1}{2}\varepsilon(1+\omega_p)\pi_t^2. \qquad (A.13)$$

In line with Benigno and Woodford (2004), we write (A.3) as $K_t = E_t\sum_{T=t}^{\infty}(\gamma\beta)^{T-t}k_{t,T}$ in which $k_{t,T} = k_T P_{t,T}^{-\varepsilon(1+\omega_p)}$. The definition of $k_{t,T}$ follows from (A.3). $P_{t,T}$ stands for the dispersion terms in $k_{t,T}$. k_T then collects the remaining terms from $k_{t,T}$. In a similar way, we can decompose F_t. z_T and X_T in (A.12) are then defined as

$$z_T = \widehat{k}_T - \widehat{f}_T, \qquad (A.14)$$

$$X_T = \widehat{k}_T + \widehat{f}_T. \qquad (A.15)$$

Using (2.10) and (2.13), k_T can be written as

$$\begin{aligned}
k_T &= \left\{\frac{C_T}{(1-\lambda)}\left[1 - \frac{\omega\lambda}{(1-H_T)(1+\omega)}\right]\right\}^{\left(1-\widetilde{\sigma}^{-1}\right)} \\
&\quad * \left[1 - \frac{H_T}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)-1}\frac{\mu\alpha\omega Y_T^{\alpha}}{(1-\tau_T)} \qquad (A.16)
\end{aligned}$$

and f_T becomes

$$\begin{aligned}
f_T &= \left\{\frac{C_T}{(1-\lambda)}\left[1 - \frac{\omega\lambda}{(1-H_T)(1+\omega)}\right]\right\}^{-\widetilde{\sigma}^{-1}} \\
&\quad * \left[1 - \frac{H_T}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}Y_T. \qquad (A.17)
\end{aligned}$$

In what follows, we also make use of the following second-order expansions. $C_t = Y_t - G_t$ can be approximated by

$$\widehat{C}_t = c^{-1}\left(\widehat{Y}_t - \widehat{G}_t\right) + \frac{1}{2}c^{-1}\left(1-c^{-1}\right)\widehat{Y}_t^2 + c^{-2}\widehat{Y}_t\widehat{G}_t + t.i.p. + O(3). \qquad (A.18)$$

c is the consumption share of income in the steady state and $t.i.p.$ refers to terms independent of policy. $(1-\tau_t)$ can be expanded as follows

$$\widehat{(1-\tau_t)} = -\omega_\tau\widehat{\tau}_t - \frac{1}{2}\frac{\omega_\tau}{(1-\overline{\tau})}\widehat{\tau}_t^2 + O(3) \qquad (A.19)$$

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where $\omega_\tau = \overline{\tau}/(1 - \overline{\tau})$. Similarly, the expansion for leisure $(1 - H_t)$ is written as

$$\widehat{(1 - H_t)} = -\omega_H \widehat{H}_t - \frac{1}{2} \frac{\omega_H}{(1 - \overline{H})} \widehat{H}_t^2 + O(3) \tag{A.20}$$

with $\omega_H = \overline{H}/(1 - \overline{H})$. From (2.18), it follows that

$$\widehat{H}_t = \alpha \widehat{Y}_t + \widehat{\delta}_t. \tag{A.21}$$

The second-order expansions of (A.7) is written as

$$\widehat{\delta}_t = \gamma \widehat{\delta}_{t-1} + \frac{1}{2} \frac{\gamma}{(1 - \gamma)} \varepsilon (1 + \omega_p)(1 + \omega_p \varepsilon) \pi_t^2 + O(3). \tag{A.22}$$

We see that the expansion does not contain linear terms in inflation and hence, when multiplied with other first-order terms, the product will be accurate to the third order. Therefore, also

$$\widehat{H}_t^2 = \alpha^2 \widehat{Y}_t^2. \tag{A.23}$$

An infinite discounted sum of (A.22) will be equal to

$$\begin{aligned} E_t \sum_{T=t}^{\infty} \beta^{T-t} \widehat{\delta}_t &= \frac{1}{2} \frac{\gamma}{(1 - \gamma)(1 - \beta\gamma)} \varepsilon (1 + \omega_p)(1 + \omega_p \varepsilon) E_t \sum_{T=t}^{\infty} \beta^{T-t} \pi_T^2 \\ &\quad + t.i.p. + O(3). \end{aligned} \tag{A.24}$$

We can also establish that

$$\begin{aligned} \left[1 - \widehat{\frac{\omega\lambda}{(1 - H_t)(1 + \omega)}} \right] &= -\Omega_H \widehat{H}_t \\ &\quad - \frac{1}{2} \Omega_H \left\{ \frac{(1 + \omega)\left(1 - \overline{H}^2\right) - \omega\lambda}{(1 - \overline{H})\left[(1 + \omega)(1 - \overline{H}) - \omega\lambda\right]} \right\} \widehat{H}_t^2 + O(3) \end{aligned}$$

and

$$\left[1 - \frac{H_t}{1 - \lambda} + \widehat{\frac{\lambda}{(1 - \lambda)(1 + \omega)}} \right] = -\chi_H \widehat{H}_t - \frac{1}{2} \chi_H (1 + \chi_H) \widehat{H}_t^2 + O(3).$$

We have defined

$$\Omega_H = \frac{\omega_H \omega\lambda}{\left[(1 + \omega)(1 - \overline{H}) - \omega\lambda\right]}$$

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and

$$\chi_H = \frac{(1 + \omega)\,\overline{H}}{(1 + \omega)\left(1 - \lambda - \overline{H}\right) + \lambda}.$$

Using the above derivations, it is now straightforward to derive the aggregate supply relationship. For z_T and X_T, we have

$$
\begin{aligned}
z_T &= \widehat{C}_T + \left[1 - \widehat{\frac{\omega\lambda}{(1 - H_T)(1 + \omega)}}\right] + (\alpha - 1)\,\widehat{Y}_t \\
&\quad - \left[1 - \frac{H_T}{1 - \lambda} + \widehat{\frac{\lambda}{(1 - \lambda)(1 + \omega)}}\right] - \widehat{(1 - \tau_t)},
\end{aligned}
\tag{A.25}
$$

$$
\begin{aligned}
X_T &= \left(1 - 2\widetilde{\sigma}^{-1}\right)\widehat{C}_T + \left(1 - 2\widetilde{\sigma}^{-1}\right)\left[1 - \widehat{\frac{\omega\lambda}{(1 - H_T)(1 + \omega)}}\right] \\
&\quad + \left[2\omega\left(1 - \widetilde{\sigma}^{-1}\right) - 1\right]\left[1 - \frac{H_T}{1 - \lambda} + \widehat{\frac{\lambda}{(1 - \lambda)(1 + \omega)}}\right] \\
&\quad + (\alpha + 1)\,\widehat{Y}_t - \widehat{(1 - \tau_t)}.
\end{aligned}
\tag{A.26}
$$

It follows that

$$
\begin{aligned}
V_t &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \zeta \left\{ z_y \widehat{Y}_T - c^{-1}\widehat{G}_T + \omega_\tau \widehat{\tau}_T \right. \\
&\quad + \frac{1}{2}\left(z_{yy} + z_y x_y\right)\widehat{Y}_T^2 + \frac{1}{2}\omega_\tau\left(\frac{1 + \overline{\tau}}{1 - \overline{\tau}}\right)\widehat{\tau}_T^2 \\
&\quad + \omega_\tau \left\{ \left(1 - \widetilde{\sigma}^{-1}\right)\left(c^{-1} - \alpha\Omega_H\right) + \alpha - \alpha\chi_H\left[\omega\left(1 - \widetilde{\sigma}^{-1}\right) - 1\right]\right\}\widehat{\tau}_T\widehat{Y}_T \\
&\quad + c^{-1}\left[c^{-1} - \left(1 - 2\widetilde{\sigma}^{-1}\right)\left(c^{-1} - \alpha\Omega_H\right)\right. \\
&\quad \left. - \alpha\left(1 - \widetilde{\sigma}^{-1}\right)\left[\chi_H(1 - \omega) + 1\right] - \widetilde{\sigma}^{-1}\right]\widehat{Y}_T\widehat{G}_T \\
&\quad \left. - c^{-1}\omega_\tau\left(1 - \widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T + \frac{1}{2}\frac{\varepsilon(1 + \omega_p)}{\zeta}\left(1 + \chi_H - \Omega_H\right)\pi_T^2 \right\} \\
&\quad + t.i.p. + O(3)
\end{aligned}
\tag{A.27}
$$

where we have used

$$z_y = c^{-1} - \alpha\Omega_H + \alpha\chi_H + \alpha - 1,\tag{A.28}$$

$$x_y = \left(1 - 2\widetilde{\sigma}^{-1}\right)\left(c^{-1} - \alpha\Omega_H\right) - \left[2\omega\left(1 - \widetilde{\sigma}^{-1}\right) - 1\right]\alpha\chi_H + \alpha + 1 \qquad \text{(A.29)}$$

and

$$
\begin{aligned}
z_{yy} &= c^{-1}\left(1 - c^{-1}\right) - \Omega_H\alpha^2 \frac{\left[(1+\omega)\left(1 - \overline{H}^2\right) - \omega\lambda\right]}{\left(1 - \overline{H}\right)\left[(1+\omega)\left(1 - \overline{H}\right) - \omega\lambda\right]} \\
&\quad + \chi_H\left(1 + \chi_H\right)\alpha^2. \qquad \text{(A.30)}
\end{aligned}
$$

To the first order, this can be written as a difference equation of the following form

$$\pi_t = \zeta\left[z_y\widehat{Y}_t - c^{-1}\widehat{G}_t + \omega_\tau\widehat{\tau}_t\right] + \beta E_t\pi_{t+1}. \qquad \text{(A.31)}$$

The fiscal solvency condition

We start with a simple flow government budget constraint

$$B_t = \left(1 + i_{t-1}\right)B_{t-1} - P_t s_t. \qquad \text{(A.32)}$$

Following Benigno and Woodford (2003), this flow budget constraint implies the following sustainability condition

$$\frac{b_{t-1}}{\Pi_t}\left(C_t^R\right)^{-\widetilde{\sigma}^{-1}}\left(1 - H_t^R\right)^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)} = E_t\sum_{T=t}^{\infty}\beta^{T-t}s_T\left(C_T^R\right)^{-\widetilde{\sigma}^{-1}}\left(1 - H_T^R\right)^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)} \qquad \text{(A.33)}$$

which requires current outstanding real liabilities to be offset by the discounted sum of future primary surpluses. We have used the Euler equation to substitute for the discount factor. We can denote the left-hand side as W_t

$$
\begin{aligned}
W_t &= \frac{b_{t-1}}{\Pi_t}\left\{\frac{C_t}{(1-\lambda)}\left[1 - \frac{\omega\lambda}{(1 - H_t)(1+\omega)}\right]\right\}^{-\widetilde{\sigma}^{-1}} \\
&\quad * \left[1 - \frac{H_t}{1 - \lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)}. \qquad \text{(A.34)}
\end{aligned}
$$

(A.33) can be written as a difference equation

$$W_t = s_t\left(C_t^R\right)^{-\widetilde{\sigma}^{-1}}\left(1 - H_t^R\right)^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)} + \beta E_t W_{t+1}. \qquad \text{(A.35)}$$

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In the steady state, it holds that

$$(1 - \beta) \overline{W} = \overline{s} \left(\overline{C}^R \right)^{-\widetilde{\sigma}^{-1}} \left(1 - \overline{H}^R \right)^{\omega \left(1 - \widetilde{\sigma}^{-1} \right)}. \tag{A.36}$$

To the first order, W_t can be approximated as

$$\widehat{W}_t = \widehat{b}_{t-1} - \pi_t - \Phi^{-1} \widehat{Y}_t + \widetilde{\sigma}^{-1} c^{-1} \widehat{G}_t + O(2) \tag{A.37}$$

with

$$\Phi^{-1} = \widetilde{\sigma}^{-1} c^{-1} - \widetilde{\sigma}^{-1} \alpha \Omega_H + \omega \alpha \left(1 - \widetilde{\sigma}^{-1} \right) \chi_H. \tag{A.38}$$

A second-order expansion of the right-hand sides of (A.33) can in turn be expressed as

$$
\begin{aligned}
s_t \left(C_t^R \right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R \right)^{\omega \left(1 - \widetilde{\sigma}^{-1} \right)} = \ & (1 - \beta) \overline{W} \left\{ 1 - \left(\widetilde{\sigma}^{-1} c^{-1} - f_H \overline{H} \alpha \right) \widehat{Y}_t + \widetilde{\sigma}^{-1} c^{-1} \widehat{G}_t \right. \\
& + \widehat{s}_t + f_H \overline{H} \widehat{\delta}_t - \left(\widetilde{\sigma}^{-1} c^{-1} - f_H \overline{H} \alpha \right) \widehat{s}_t \widehat{Y}_t \\
& + \widetilde{\sigma}^{-1} c^{-1} \widehat{s}_t \widehat{G}_t - \frac{1}{2} \left[\widetilde{\sigma}^{-1} c^{-1} - \widetilde{\sigma}^{-1} \left(\widetilde{\sigma}^{-1} + 1 \right) c^{-2} \right. \\
& \left. - f_H \overline{H} \alpha^2 - f_{HH} \overline{H}^2 \alpha^2 + 2 \widetilde{\sigma}^{-1} c^{-1} f_H \overline{H} \alpha \right] \widehat{Y}_t^2 \\
& \left. - \widehat{Y}_t \widehat{G}_t \left[\widetilde{\sigma}^{-1} \left(\widetilde{\sigma}^{-1} + 1 \right) c^{-2} - \widetilde{\sigma}^{-1} c^{-1} f_H \overline{H} \alpha \right] \right\} \\
& + t.i.p. + O(3).
\end{aligned}
\tag{A.39}
$$

We have used

$$(1 - \beta) \overline{W} f_H = \frac{\partial \left[(1 - \beta) \overline{W} / \overline{\Delta} \right]}{\partial \overline{H}}$$

and

$$(1 - \beta) \overline{W} f_{HH} = \frac{\partial^2 \left[(1 - \beta) \overline{W} / \overline{\Delta} \right]}{\partial \overline{H}^2}.$$

In an economy with tax on wage income, the total tax revenues are going to be given by

$$
\begin{aligned}
T_t &= \tau_t \frac{W_t}{P_t} H_t \\
&= \omega \frac{\tau_t}{1 - \tau_t} \frac{H_t}{1 - H_t} C_t.
\end{aligned}
\tag{A.40}
$$

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In the steady state, total tax revenues equal

$$\overline{T} = \omega\omega_\tau\omega_H\overline{C}. \tag{A.41}$$

$s_t = T_t - G_t$ can then be approximated by

$$
\begin{aligned}
\widehat{s}_t &= \frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_t + d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\widehat{Y}_t - \left(d_\tau^{-1}c^{-1} + d^{-1}\right)\widehat{G}_t \\
&\quad + \frac{d_\tau^{-1}}{1-\overline{H}}\widehat{\delta}_t + \frac{1}{2}d_\tau^{-1}\left[c^{-1} + \frac{(1+2\omega_H)\,\alpha^2}{1-\overline{H}} + \frac{\alpha c^{-1}}{1-\overline{H}}\right]\widehat{Y}_t^2 \\
&\quad + \frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_t^2 + \frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\widehat{\tau}_t\widehat{Y}_t \\
&\quad - \frac{d_\tau^{-1}}{1-\overline{\tau}}c^{-1}\widehat{\tau}_t\widehat{G}_t - \frac{d_\tau^{-1}}{1-\overline{H}}\alpha c^{-1}\widehat{Y}_t\widehat{G}_t \\
&\quad + t.i.p. + O(3). \tag{A.42}
\end{aligned}
$$

We have defined $d_\tau = \overline{s}/\overline{T}$ and $d = \overline{s}/\overline{Y}$. Using this in (A.39) yields

$$
\begin{aligned}
s_t\left(C_t^R\right)^{-\widetilde{\sigma}^{-1}}\left(1-H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} &= (1-\beta)\,\overline{W}\left\{1 + \frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_t\right. \\
&\quad + \left[f_H\overline{H}\alpha - \widetilde{\sigma}^{-1}c^{-1} + d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\right]\widehat{Y}_t \\
&\quad - \left(d_\tau^{-1}c^{-1} + d^{-1} - \widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_t + \left(\frac{d_\tau^{-1}}{1-\overline{H}} + f_H\overline{H}\right)\widehat{\delta}_t \\
&\quad + \frac{1}{2}f_{YY}\widehat{Y}_t^2 + \frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_t^2 \\
&\quad + \frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}} + c^{-1} - \widetilde{\sigma}^{-1}c^{-1} + f_H\overline{H}\alpha\right)\widehat{\tau}_t\widehat{Y}_t \\
&\quad \left. - \frac{d_\tau^{-1}c^{-1}}{1-\overline{\tau}}\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_t\widehat{G}_t - f_{YG}\widehat{Y}_t\widehat{G}_t\right\} \\
&\quad t.i.p. + O(3) \tag{A.43}
\end{aligned}
$$

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with

$$
\begin{aligned}
f_{YY} &= d_\tau^{-1}\left[c^{-1} + \frac{(1+2\omega_H)\,\alpha^2}{1-\overline{H}} + \frac{\alpha c^{-1}}{1-\overline{H}}\right] - \widetilde{\sigma}^{-1}c^{-1} + \widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2} + f_H\overline{H}\alpha^2 \\
&\quad + f_{HH}\overline{H}^2\alpha^2 - 2\widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha - 2d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\left(\widetilde{\sigma}^{-1}c^{-1} - f_H\overline{H}\alpha\right)
\end{aligned}
$$
(A.44)

and

$$
\begin{aligned}
f_{YG} &= \frac{d_\tau^{-1}}{1-\overline{H}}\alpha c^{-1} - \left(\widetilde{\sigma}^{-1}c^{-1} - f_H\overline{H}\alpha\right)\left(d_\tau^{-1}c^{-1} + d^{-1}\right) \\
&\quad - \widetilde{\sigma}^{-1}c^{-1}d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right) + \widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2} - \widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha.
\end{aligned}
$$
(A.45)

Plugging (A.43) back into (A.33), and using (A.24)

$$
\begin{aligned}
\widehat{W}_t &= (1-\beta)\,E_t\sum_{T=t}^{\infty}\beta^{T-t}\Bigg\{\left[f_H\overline{H}\alpha - \widetilde{\sigma}^{-1}c^{-1} + d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\right]\widehat{Y}_T \\
&\quad + \frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_T - \left(d_\tau^{-1}c^{-1} + d^{-1} - \widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_T + \frac{1}{2}f_{YY}\widehat{Y}_T^2 \\
&\quad + \frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_T^2 + \frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}} + c^{-1} - \widetilde{\sigma}^{-1}c^{-1} + f_H\overline{H}\alpha\right)\widehat{\tau}_T\widehat{Y}_T \\
&\quad - \frac{d_\tau^{-1}c^{-1}}{1-\overline{\tau}}\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T - f_{YG}\widehat{Y}_T\widehat{G}_T \\
&\quad \frac{1}{2}\left(\frac{d_\tau^{-1}}{1-\overline{H}} + f_H\overline{H}\right)\frac{\varepsilon(1+\omega_p)}{\zeta}\pi_T^2\Bigg\} + t.i.p. + O(3).
\end{aligned}
$$
(A.46)

With (A.37) on the left-hand side, this can be written with first-order accuracy as

$$
\begin{aligned}
\widehat{b}_{t-1} - \pi_t - \Phi^{-1}\widehat{Y}_t + \widetilde{\sigma}^{-1}c^{-1}\widehat{G}_t &= (1-\beta)\Big[f_y\widehat{Y}_t + f\tau\widehat{\tau}_t \\
&\quad - \left(d_\tau^{-1}c^{-1} + d^{-1} - \widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_t\Big] \\
&\quad + \beta E_t\left[\widehat{b}_t - \pi_{t+1} - \Phi^{-1}\widehat{Y}_{t+1} + \widetilde{\sigma}^{-1}c^{-1}\widehat{G}_{t+1}\right].
\end{aligned}
$$
(A.47)

The definitions of f_y and $f\tau$ follow from (A.46).

Approximation to utility

The weighted average of period utilities can be written in terms of aggregate variables as follows

$$
\begin{aligned}
u_t \;=\; & \frac{C_t^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}} \left\{ \lambda \left(1 - H_t\right)^{\widetilde{\sigma}^{-1}-1} \left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)} \right. \\
& + (1-\lambda)^{\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega \lambda}{\left(1 - H_t\right)\left(1+\omega\right)}\right]^{1-\widetilde{\sigma}^{-1}} \\
& \times \left. \left[1 - \frac{H_t}{1-\lambda} + \frac{\lambda}{\left(1-\lambda\right)\left(1+\omega\right)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \right\}.
\end{aligned}
$$

$$(A.48)$$

The second order approximation to the discounted sum of expected utilities given by this expression can be written as

$$
\begin{aligned}
U_t \;=\; & \overline{C}^{1-\widetilde{\sigma}^{-1}} E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \Theta_Y \widehat{Y}_T + \right. \\
& + \frac{1}{2} \left[Kc^{-1}\left(1 - \widetilde{\sigma}^{-1}c^{-1}\right) + L\overline{H}\alpha\left(\alpha + 2\left(1 - \widetilde{\sigma}^{-1}\right)c^{-1}\right) + M\overline{H}^2\alpha^2 \right] \widehat{Y}_T^2 \\
& + \left[K\widetilde{\sigma}^{-1}c^{-2} - L\overline{H}\alpha c^{-1}\left(1 - \widetilde{\sigma}^{-1}\right)\right] \widehat{Y}_T \widehat{G}_T + \frac{1}{2} L\overline{H}\frac{\varepsilon\left(1+\omega_p\right)}{\zeta}\pi_T^2 \right\} \\
& + t.i.p. + O(3)
\end{aligned}
$$

$$(A.49)$$

where $\Theta_Y = \left(Kc^{-1} + L\overline{H}\alpha\right)$ with

$$
\begin{aligned}
K \;=\; & \lambda\left(1 - \overline{H}\right)^{\widetilde{\sigma}^{-1}-1} \left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)} + \\
& (1-\lambda)^{\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{\left(1 - \overline{H}\right)\left(1+\omega\right)}\right]^{1-\widetilde{\sigma}^{-1}} \left[1 - \frac{\overline{H}}{1-\lambda} + \frac{\lambda}{\left(1-\lambda\right)\left(1+\omega\right)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}
\end{aligned}
$$

$$(A.50)$$

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$$
\begin{aligned}
L \;=\; & \lambda \left(1 - \overline{H}\right)^{\widetilde{\sigma}^{-1}-2} \left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)} \\[2mm]
& - \left[1 - \frac{\overline{H}}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \left[1 - \frac{\omega\lambda}{\left(1-\overline{H}\right)(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}} \\[2mm]
& \times \left\{ \frac{\lambda\omega(1-\lambda)^{\widetilde{\sigma}^{-1}}}{(1+\omega)\left(1-\overline{H}\right)^2} + \frac{\omega(1-\lambda)^{\widetilde{\sigma}^{-1}}\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]}{\left(1-\overline{H}\right)\left[\left(1-\lambda-\overline{H}\right)(1+\omega)+\lambda\right]} \right\} \quad (A.51)
\end{aligned}
$$

and

$$
\begin{aligned}
M \;=\; & \lambda \left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)} \left(2-\widetilde{\sigma}^{-1}\right)\left(1-\overline{H}\right)^{\widetilde{\sigma}^{-1}-3} \\[2mm]
& + \left[1 - \frac{\overline{H}}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \left[1 - \frac{\omega\lambda}{\left(1-\overline{H}\right)(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}} \\[2mm]
& \times \frac{\omega(1-\lambda)^{\widetilde{\sigma}^{-1}}}{\left(1-\overline{H}\right)} \left\{ \frac{2\omega\lambda\left(1-\widetilde{\sigma}^{-1}\right)}{\left(1-\overline{H}\right)\left[\left(1-\lambda-\overline{H}\right)(1+\omega)+\lambda\right]} - \frac{2\lambda}{(1+\omega)\left(1-\overline{H}\right)^2} \right. \\[2mm]
& - \frac{\omega\widetilde{\sigma}^{-1}\lambda^2}{(1+\omega)\left(1-\overline{H}\right)^2\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]} \\[2mm]
& \left. + \frac{\left[\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right](1+\omega)\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]}{\left[\left(1-\lambda-\overline{H}\right)(1+\omega)+\lambda\right]^2} \right\}. \quad (A.52)
\end{aligned}
$$

The policy objective function

The approximation (A.49) contains a linear term in one of the endogenous variables, which means that policies aiming at stabilization of deviations of fundamentals from their steady-state values not only affect the variation of the variables but also have significant level effects. However, as also mentioned in the paper, (A.49) will not serve as a correct, second-order-accurate welfare-ranking criterion for optimal policies obtained as a solution to a system of linear structural equations (derived above). To obtain a second-order-accurate welfare ranking criterion, one has to use second-order approximations to structural equations to substitute out the linear term from (A.49). This way, a policy objective expressed in second-order terms only is obtained. The level-effects are preserved in an implicit form.

The second-order approximations to the fiscal solvency condition and the aggregate supply relationship can be written as

$$
\begin{aligned}
\frac{\widehat{W}_t}{(1-\beta)} \;=\; & E_t \sum_{T=t}^{\infty} \beta^{T-t} \Bigg\{ f_y \widehat{Y}_T + \frac{d_\tau^{-1}}{1-\overline{\tau}} \widehat{\tau}_T - \left(d_\tau^{-1} c^{-1} + d^{-1} - \widetilde{\sigma}^{-1} c^{-1} \right) \widehat{G}_T \\
& + \frac{1}{2} f_{YY} \widehat{Y}_T^2 + \frac{1}{2} \frac{d_\tau^{-1}}{1-\overline{\tau}} \left(1 + 2\omega_\tau \right) \widehat{\tau}_T^2 \\
& + \frac{d_\tau^{-1}}{1-\overline{\tau}} \left(\frac{\alpha}{1-\overline{H}} + c^{-1} - \widetilde{\sigma}^{-1} c^{-1} + f_H \overline{H} \alpha \right) \widehat{\tau}_T \widehat{Y}_T \\
& - \frac{d_\tau^{-1} c^{-1}}{1-\overline{\tau}} \left(1 - \widetilde{\sigma}^{-1} \right) \widehat{\tau}_T \widehat{G}_T - f_{YG} \widehat{Y}_T \widehat{G}_T \\
& \frac{1}{2} \left(\frac{d_\tau^{-1}}{1-\overline{H}} + f_H \overline{H} \right) \frac{\varepsilon \left(1 + \omega_p \right)}{\zeta} \pi_T^2 \Bigg\} + t.i.p. + O(3), \qquad (A.53)
\end{aligned}
$$

$$
\begin{aligned}
\frac{V_t}{\zeta} \;=\; & E_t \sum_{T=t}^{\infty} \beta^{T-t} \Bigg\{ z_y \widehat{Y}_T - c^{-1} \widehat{G}_T + \omega_\tau \widehat{\tau}_T \\
& + \frac{1}{2} \left(z_{yy} + z_y x_y \right) \widehat{Y}_T^2 + \frac{1}{2} \omega_\tau \left(\frac{1+\overline{\tau}}{1-\overline{\tau}} \right) \widehat{\tau}_T^2 \\
& + \omega_\tau \left\{ \left(1 - \widetilde{\sigma}^{-1} \right) \left(c^{-1} - \alpha \Omega_H \right) + \alpha - \alpha \chi_H \left[\omega \left(1 - \widetilde{\sigma}^{-1} \right) - 1 \right] \right\} \widehat{\tau}_T \widehat{Y}_T \\
& + c^{-1} \left[c^{-1} - \left(1 - 2\widetilde{\sigma}^{-1} \right) \left(c^{-1} - \alpha \Omega_H \right) \right. \\
& \left. - \alpha \left(1 - \widetilde{\sigma}^{-1} \right) \left[\chi_H \left(1 - \omega \right) + 1 \right] - \widetilde{\sigma}^{-1} \right] \widehat{Y}_T \widehat{G}_T \\
& - c^{-1} \omega_\tau \left(1 - \widetilde{\sigma}^{-1} \right) \widehat{\tau}_T \widehat{G}_T + \frac{1}{2} \frac{\varepsilon \left(1 + \omega_p \right)}{\zeta} \left(1 + \chi_H - \Omega_H \right) \pi_T^2 \Bigg\} \\
& + t.i.p. + O(3). \qquad (A.54)
\end{aligned}
$$

The task is to find a linear combination of these two equations such that contains coefficients at linear terms of endogenous variables equal to those in (A.49), which will enable us to substitute out the linear term in (A.49). We are therefore looking for a vector of coefficients $\mathbf{v}' = (v_1, v_2)$ such that satisfy the following system of equations

$$
\begin{pmatrix} f_y & z_y \\ \frac{d_\tau^{-1}}{1-\overline{\tau}} & \omega_\tau \end{pmatrix} \mathbf{v} = \begin{pmatrix} \Theta_Y \\ 0 \end{pmatrix}.
$$

We find that

$$\mathbf{v} = \begin{pmatrix} -\frac{\Theta_Y}{\Omega}\omega_\tau\left(1-\overline{\tau}\right) \\ \frac{\Theta_Y}{\Omega}d_\tau^{-1} \end{pmatrix} \tag{A.55}$$

with $\Omega = z_y d_\tau^{-1} - \omega_\tau\left(1-\overline{\tau}\right)f_y$. It therefore holds that

$$
\begin{aligned}
E_t\sum_{T=t}^{\infty}\beta^{T-t}\Theta_Y\widehat{Y}_T &= E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\mathbf{v}'\boldsymbol{\xi}_Y\widehat{Y}_T + \mathbf{v}'\boldsymbol{\xi}_\tau\widehat{\tau}_{L,T}\right\} \\
&= -E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\frac{1}{2}\left[v_1 f_{YY} + v_2\left(z_{yy}+z_y x_y\right)\right]\widehat{Y}_T^2\right. \\
&\quad - \left(v_1 f_{YG} - v_2 x_{YG}\right)\widehat{Y}_T\widehat{G}_T \\
&\quad \left.+\frac{1}{2}\frac{\varepsilon\left(1+\omega_p\right)}{\zeta}\left[v_1\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)+v_2\left(1+\chi_H-\Omega_H\right)\right]\right\}\pi_T^2 \\
&\quad +v_1\frac{\widehat{W}_t}{\left(1-\beta\right)}+v_2\frac{V_t}{\zeta}+t.i.p.+O\left(3\right)
\end{aligned}
\tag{A.56}
$$

where $\boldsymbol{\xi}_Y' = \left(f_y, z_y\right)$ and $\boldsymbol{\xi}_\tau' = \left(\frac{d_\tau^{-1}}{1-\overline{\tau}}, \omega_\tau\right)$. The definition of x_{YG} follows from the (A.54). We can now substitute this into (A.49) to obtain

$$
\begin{aligned}
U_t &= -\overline{C}^{1-\widetilde{\sigma}^{-1}}E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\frac{1}{2}q_y\widehat{Y}_T^2 - q_{YG}\widehat{Y}_T\widehat{G}_T + \frac{1}{2}q_\pi\pi_T^2\right\} \\
&\quad +v_1\overline{C}^{1-\widetilde{\sigma}^{-1}}\frac{\widehat{W}_t}{\left(1-\beta\right)}+v_2\overline{C}^{1-\widetilde{\sigma}^{-1}}\frac{V_t}{\zeta}+t.i.p.+O\left(3\right).
\end{aligned}
\tag{A.57}
$$

The coefficients are defined as follows

$$
\begin{aligned}
q_y &= \frac{\Theta_Y}{\Omega}d_\tau^{-1}\left(z_{yy}+z_y x_y\right) - \frac{\Theta_Y}{\Omega}\overline{\tau}f_{YY} - Kc^{-1}\left(1-\widetilde{\sigma}^{-1}c^{-1}\right) \\
&\quad -L\overline{H}\alpha\left(\alpha+2\left(1-\widetilde{\sigma}^{-1}\right)c^{-1}\right)-M\overline{H}^2\alpha^2,
\end{aligned}
\tag{A.58}
$$

$$q_{YG} = K\widetilde{\sigma}^{-1}c^{-2} - L\overline{H}\alpha c^{-1}\left(1-\widetilde{\sigma}^{-1}\right) - \frac{\Theta_Y}{\Omega}\overline{\tau}f_{YG} - \frac{\Theta_Y}{\Omega}d_\tau^{-1}x_{YG} \tag{A.59}$$

and

$$q_\pi = \frac{\varepsilon\left(1+\omega_p\right)}{\zeta}\left[\frac{\Theta_Y}{\Omega}\left(1-\overline{\tau}\right)\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)-L\overline{H}\right]. \tag{A.60}$$

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(A.57) can also be expressed as

$$U_t = -\overline{C}^{1-\tilde{\sigma}^{-1}} E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \frac{1}{2} q_y \left(\widehat{Y}_T - \widehat{Y}_T^* \right)^2 + \frac{1}{2} q_\pi \pi_T^2 \right\}$$
$$+ v_1 \overline{C}^{1-\tilde{\sigma}^{-1}} \frac{\widehat{W}_t}{(1-\beta)} + v_2 \overline{C}^{1-\tilde{\sigma}^{-1}} \frac{V_t}{\zeta} + t.i.p. + O(3) \qquad \text{(A.61)}$$

where we have defined

$$\widehat{Y}_T^* = \frac{q_{YG}}{q_y} \widehat{G}_T. \qquad \text{(A.62)}$$

$\widehat{Y}_T^*$ represents the 'target' deviation of output from its steady state mentioned in the main text. The linear terms in the second line represent the welfare effect of the policy in the initial period t. This is a transitory component. Benigno and Woodford (2003) explain why this component can be treated as predetermined in what they call the second stage of the Ramsey problem.[1].

We can now re-write all our structural equations in terms of the welfare-relevant output gap. The price aggregate supply relationship (A.31) becomes

$$\pi_t = \kappa y_t + \chi_\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) + \beta E_t \pi_{t+1} \qquad \text{(A.63)}$$

with $y_t = \widehat{Y}_T - \widehat{Y}_T^*$, $\kappa = \zeta z_y$ and $\chi_\tau = \zeta \omega_\tau$ and $\widehat{\tau}_t^* = \frac{z_y}{\omega_\tau} \left(\frac{c^{-1}}{z_y} - \frac{q_{YG}}{q_y} \right) \widehat{G}_t$.

The fiscal solvency condition becomes

$$\widehat{b}_{t-1} - \pi_t - \Phi^{-1} y_t + \varphi_t = (1-\beta) [f_y y_t + f\tau (\widehat{\tau}_t - \widehat{\tau}_t^*)]$$
$$+ \beta E_t \left[\widehat{b}_t - \pi_{t+1} - \Phi^{-1} y_{t+1} + \varphi_{t+1} \right] \qquad \text{(A.64)}$$

where

$$\varphi_t = \left(\tilde{\sigma}^{-1} c^{-1} - \Phi^{-1} \frac{q_{YG}}{q_y} \right) \widehat{G}_t$$
$$+ (1-\beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ d_\tau^{-1} c^{-1} + d^{-1} - \tilde{\sigma}^{-1} c^{-1} \right.$$
$$\left. - f_y \frac{q_{YG}}{q_y} - \frac{f\tau}{\omega_\tau} \left(c^{-1} - z_y \frac{q_{YG}}{q_y} \right) \right\} \widehat{G}_T. \qquad \text{(A.65)}$$

---

[1]See also the discussion in Woodford (2003, Chapter 6).

The IS relationship

The Euler equation can be written in terms of the aggregate variables as follows

$$\beta \frac{E_t \left(C_{t+1}\right)^{-\tilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{(1-H_{t+1})(1+\omega)}\right]^{-\tilde{\sigma}^{-1}} \left[1 - \frac{H_{t+1}}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\tilde{\sigma}^{-1}\right)}}{C_t^{-\tilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{(1-H_t)(1+\omega)}\right]^{-\tilde{\sigma}^{-1}} \left[1 - \frac{H_t}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\tilde{\sigma}^{-1}\right)}} \frac{(1+i_t)}{(1+E_t\pi_{t+1})} = 1.$$

$$(\text{A.66})$$

A straightforward log-linearization of this relationship yields

$$\Phi^{-1}\widehat{Y}_t = \Phi^{-1}E_t\widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t\pi_{t+1}\right) - \tilde{\sigma}^{-1}c^{-1}\left(E_t\widehat{G}_{t+1} - \widehat{G}_t\right). \qquad (\text{A.67})$$

In this, we have assumed that $\widehat{\delta}_{-1} = 0$ or equivalently, that t is large enough so that $\gamma^t\widehat{\delta}_{-1}$ is insignificantly different from 0. The slope coefficient Φ^{-1} has been defined above. Rewriting (A.67) in terms of the welfare-relevant output gap, we obtain

$$y_t = E_t y_{t+1} - \Phi\left(\widehat{i}_t - E_t\pi_{t+1}\right) + \left(\frac{q_{YG}}{q_y} - \Phi\tilde{\sigma}^{-1}c^{-1}\right)\left(E_t\widehat{G}_{t+1} - \widehat{G}_t\right). \qquad (\text{A.68})$$

For the persistent shock assumed in the paper, we obtain the equation in the text with $\widehat{r}_t^* = \left(\tilde{\sigma}^{-1}c^{-1} - \Phi^{-1}\frac{q_{YG}}{q_y}\right)(1 - \rho_G)\widehat{G}_t$.

The steady state

The values for the optimal steady state can be derived as follows. In the steady state, prices are stable and hence there is no dispersion. (A.2) becomes

$$1 = \frac{\mu\alpha\omega\overline{C}^R\overline{Y}^{\alpha-1}}{(1-\overline{\tau})\left(1-\overline{H}^R\right)} \qquad (\text{A.69})$$

which can also be written in terms of aggregate variables as follows

$$1 = \frac{\mu\alpha\omega c \left[1 - \frac{\omega\lambda}{(1-\overline{Y}^\alpha)(1+\omega)}\right]\overline{Y}^\alpha}{(1-\lambda)(1-\overline{\tau})\left[1 - \frac{\overline{Y}^\alpha}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]} \qquad (\text{A.70})$$

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where we have used $c = \overline{C}/\overline{Y}$ and $\overline{H} = \overline{Y}^\alpha$. This relationship defines the steady-state level of output. It is easy to show that this relationship yields two solutions for the steady-state output, one of which represents a special case with the Ricardian agents consuming no leisure ($\overline{H}^R = 1$) so that $\overline{H} = \frac{1+\omega-\omega\lambda}{1+\omega}$. This implies a corner solution case, a case of zero consumption for Ricardians in the steady state. A positive deviation from this steady state then implies an infinite increase in utility for Ricardian agents and for the whole economy too. We therefore concentrate on the other solution which can be written as

$$\overline{Y} = \left[\frac{-b_Y - \sqrt{b_Y^2 - 4a_Y c_Y}}{2a_Y} \right]^{\frac{1}{\alpha}} \tag{A.71}$$

where

$$a_Y = (1+\omega)(1-\overline{\tau}+\mu\alpha\omega c),$$
$$b_Y = \mu\alpha\omega c\,(\omega\lambda - 1 - \omega) - (1-\overline{\tau})(1+\omega)(2-\lambda) - (1-\overline{\tau})\lambda$$

and

$$c_Y = (1-\overline{\tau})[(1-\lambda)(1+\omega)+\lambda].$$

(A.71) can be shown to be equal to

$$\overline{Y} = \left[\frac{1-\overline{\tau}}{\mu\alpha\omega c + 1 - \overline{\tau}} \right]^{\frac{1}{\alpha}}$$

which is independent of λ.

Next, (2.7) implies that in the steady state

$$\overline{R} = \overline{I} = \frac{1}{\beta}. \tag{A.72}$$

$\overline{R} = (1+\overline{r})$ and $\overline{I} = (1+\overline{i})$, $\overline{r}$ and $\overline{i}$ are real and nominal interest rates, respectively. Finally, (A.35) implies

$$
\begin{aligned}
(1-\beta)\overline{W} &= \overline{s}\left(\overline{C}^R\right)^{-\widetilde{\sigma}^{-1}}\left(1-\overline{H}^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \\
&= \overline{s}\left(\frac{\overline{C}}{1-\lambda}\right)^{-\widetilde{\sigma}^{-1}}\left[1-\frac{\omega\lambda}{(1-\overline{H})(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}} \\
&\quad \times \left[1-\frac{\overline{H}}{1-\lambda}+\frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}
\end{aligned}
$$

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which, given the definition of $\overline{W}$, gives us

$$\overline{s} = (1 - \beta)\,\overline{b}. \tag{A.73}$$

Targeting rule coefficients

The coefficients in (3.5) are defined as follows

$$\omega_\phi = q_\pi^{-1}\left[1 + \frac{(1 - \beta)\,f_\tau}{\chi_\tau}\right],$$

$$m_\phi = q_y^{-1}\left[(1 - \beta)\,f_y + \Phi^{-1} - \frac{(1 - \beta)\,f_\tau\kappa}{\chi_\tau}\right]$$

and

$$n_\phi = -\frac{\Phi^{-1}}{q_y}.$$

B. Calibrated values of coefficients

In the second section of the annex, we plot some of the coefficients defined in the previous section as a function of the population share of non-Ricardian agents under baseline calibration described in Section 4.1 of the paper, assuming a non-persistent shock. We also provide a brief comment on the relative weight of output gap stabilization in the policy objective under different calibrations of structural parameters.

The coefficients in (3.1) turn out to be independent of lambda. This follows from the fact that the equilibrium real wage rate and hence also marginal cost in our economy only depends on aggregate variables, as implicitly defined in (2.11). The coefficient f_y is convex in the share of non-Ricardian agents, reflecting the fact that the sensitivity of marginal utility of consumption of Ricardian agents—the unit of account in the equation—to output increases in λ. The costliness of output (gap) volatility rises as the first-order welfare-gains from lower volatility rise with λ, mainly due to the shape of the Ricardian agents' labour supply function, as discussed in section 4 of the paper. The link between q_y, the target deviation, fiscal stress, and the efficient tax and interest rate response is discussed extensively in sections 4 and 5. The costliness of inflation volatility depends on similar level-effect considerations as is the case with output gap volatility. Moreover, it is inversely proportional to the slope of the aggregate supply relationship, which is
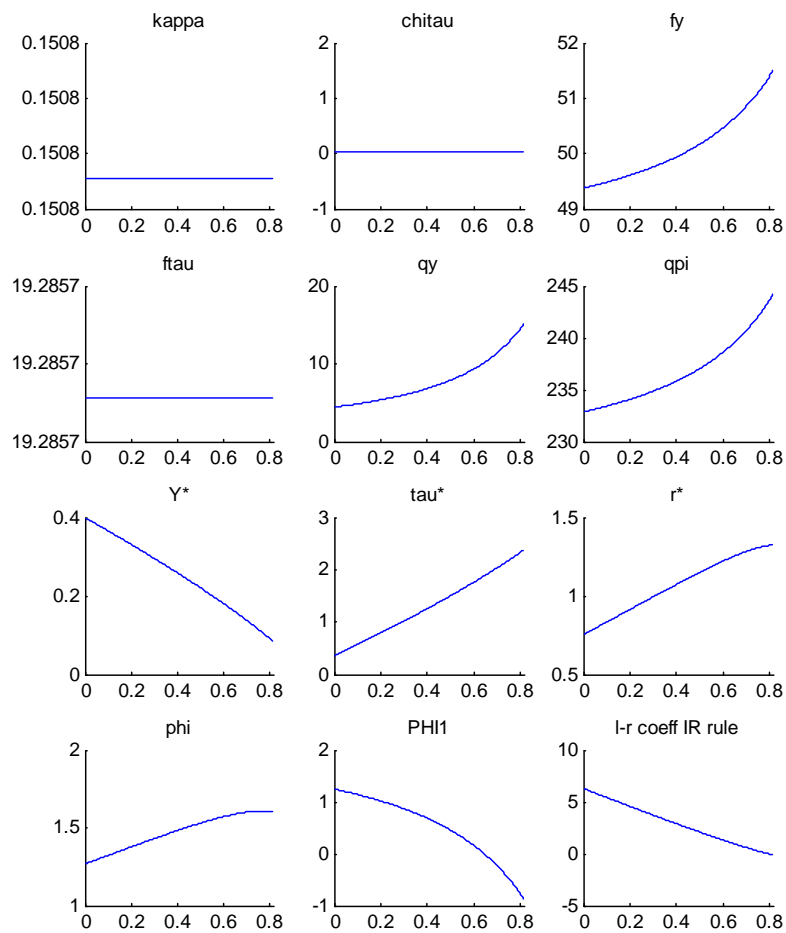
Figure B.1: Coefficient values as a function of the share of non-Ricardian agents in the economy
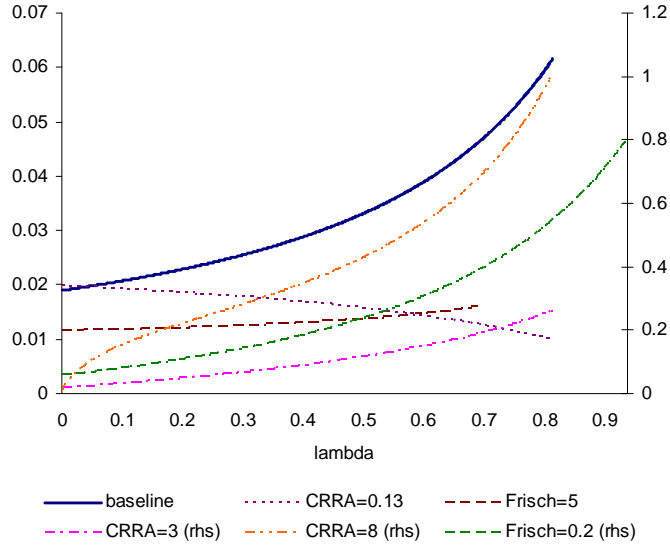
Figure B.2: Relative weight of output gap stabilization as a function of λ

in turn inversely related to the degree of price flexibility. The link between the slope of the 'IS' relationship and the long-run response coefficient in the interest rate reaction function is covered in Section 5 of the paper.

As regards the relative weight of output gap stabilization in the policy objective, Figure B.2 shows that it is generally very low for low levels of lambda. It falls even further with λ, if risk aversion is low. However, it rises with λ in all other considered cases. For large degrees of risk aversion or inelastic labour supply, coupled with a large share of λ, the importance of output gap stabilization is comparable with that of stabilizing inflation volatility.

C. Links with the literature on consumer behaviour

The model presented in the paper can be linked to the existing literature on consumer behaviour as follows. One way to write the log-linearized version of (2.7) is

$$E_t \Delta \widehat{C}_{t+1} = \widetilde{\sigma} \left(\widehat{i}_t - E_t \pi_{t+1} \right) + \Phi_Y E_t \Delta \widehat{Y}_{t+1}. \tag{C.1}$$

Here, Δ denotes change from previous period. The coefficient Φ_Y is of particular importance here. It is defined

$$\Phi_Y = \widetilde{\sigma}\left[\widetilde{\sigma}^{-1}\alpha\Omega_H - \omega\alpha\left(1 - \widetilde{\sigma}^{-1}\right)\chi_H\right]$$

in which $\overline{H}$ denotes steady-state aggregate labour supply.

Notice that with $\widetilde{\sigma}^{-1} = 1$ and $\lambda = 0$, which corresponds to the case of a log-linear specification of individual utility and no liquidity constraints, $\Phi_Y = 0$ and (C.1) breaks down to a standard intertemporal IS relationship. With $\widetilde{\sigma}^{-1} = 1$ and $\lambda > 0$, we have $\Phi_Y = \alpha\Omega_H$, which will be positive as long as $\overline{H} < 1 - \omega\lambda/(1+\omega)$ or $\overline{H}^R < 1$, which holds for all parameter values when $\lambda \in [0, \overline{\lambda}]$. This case corresponds to Campbell and Mankiw's (1989) explanation for why researchers estimated a positive coefficient at expected change in aggregate income in (C.1).[2] According to this theory, expected changes in aggregate income are associated with contemporaneous changes in consumption due to the presence of liquidity-constrained agents. By contrast, consider the case when $\widetilde{\sigma}^{-1} \neq 1$ and $\lambda = 0$. In such a case, $\Phi_Y = -\omega\alpha\left(1 - \widetilde{\sigma}^{-1}\right)\widetilde{\sigma}\chi_H$, which can clearly be positive if $\chi_H > 0$—which is in turn positive under the same condition as Ω_H—and when $\widetilde{\sigma}^{-1} > 1$. This would correspond to the 'non-separability in preferences' story put forward by Basu and Kimball (2002) as an alternative to Campbell and Mankiw (1989). Interestingly, Basu and Kimball (2002) estimated $\widetilde{\sigma}$ to be low, perhaps around one third, enhancing the consistency with the algebraic analysis presented here. Obviously, a positive coefficient Φ_Y might result as a combination of the two effects too.

D. Inverted aggregate demand relationship

Under certain circumstances, the slope coefficient in the aggregate demand relationship (5.2) becomes negative. Here we explain in more detail why this phenomenon arises in our model.

To uncover what we shall refer to as the 'principal cause' of an inverted aggregate demand schedule in our model, which is the explanation put forward by Bilbiie (2008), let us assume $\widetilde{\sigma}^{-1} = 1$. Then, the inverse of the slope of the aggregate demand relationship reduces to

---

[2]See Hall (1978), Flavin (1981) and Zeldes (1989) for earlier empirical evidence questioning the permanent income hypothesis under which no contemporaneous effects of expected changes in income on consumption should be observed.

$$\Phi^{-1} = \widetilde{\sigma}^{-1} c^{-1} - \widetilde{\sigma}^{-1} \alpha \Omega_H. \tag{D.1}$$

In order for this to become negative, corresponding to an upward-sloping 'IS' schedule, $\Omega_H > c^{-1}/\alpha$. It is easy to show that this is same as to say that

$$\frac{\lambda C^{NR}}{(1-\lambda)\, C^R} > \frac{c^{-1}}{\alpha \omega_H} \tag{D.2}$$

in which ω_H is the inverse of the Frisch elasticity of labour supply. Non-Ricardian agents must consume a sufficiently large share of the total pie. To add more intuition, it is easy to show that (2.13) can also be written as

$$C_t^R = \frac{C_t}{(1-\lambda)} \left[1 - \lambda \frac{C_t^{NR}}{C_t} \right] \tag{D.3}$$

from which it follows that

$$\widehat{C}_t^R = \widehat{C}_t - \frac{\lambda C^{NR}}{(1-\lambda)\, C^R} \left(\widehat{\frac{C_t^{NR}}{C_t}} \right). \tag{D.4}$$

The ratio of per capita non-Ricardian to aggregate consumption is positively related to total hours worked, with the elasticity ω_H, and hence also net real wages. This is intuitive, as a change in current total income would fully translate into a rise in the consumption of non-Ricardian agents but would only slightly change the consumption habits of Ricardian agents, as they look at lifetime income, and hence, average and total consumption would change less. Given a change in aggregate consumption induced by a change in output and hours worked, for a sufficiently large share of non-Ricardian agents, consumption of Ricardian agents must change in the opposite direction in order to bring about the rise in the ratio of per capita non-Ricardian to aggregate consumption required by (D.4). From a forward integrated version of (2.5), we can infer that the only way the consumption of Ricardian agents can go in the opposite direction as their wage earnings (that is the consumption of non-Ricardian agents), is when distributed profits counteract the effect of the change in wage income. In the end then, a fall in consumption of Ricardian agents when output rises means that higher output will be associated with higher marginal utility of consumption. In terms of the Euler equation, if the response of the Ricardians to a rise in the real interest rate is to re-allocate consumption from today to tomorrow, aggregate income would move

in the opposite direction due to rising profits. It would be rising at the same time as real interest rate is rising, which is at odds with conventional understanding of the aggregate demand relationship.

There is, however, a further element present in our analysis, which follows from non-separability of preferences. To see this, consider the case when $\widetilde{\sigma}^{-1} \neq 1$ but $\lambda = 0$. Totally differentiating marginal utility of consumption, we get

$$du_C = u_{CC}dC + u_{CH}dH \tag{D.5}$$

in which $u_{CC} < 0$ but, as noted in section 2.1 of the paper, u_{CH} changes sign depending on whether $\widetilde{\sigma}^{-1}$ is greater or smaller than 1. Once again, the Euler equation tells us that agents adjust their behaviour in response to changes in the real interest rate until an appropriate intertemporal ratio of marginal utilities is attained. With separable preferences, this normally implies that a rise in the real interest rate requires a fall in the ratio of marginal utilities, which are decreasing in consumption and output, resulting in a re-allocation of consumption from today to tomorrow. Current consumption and production are thus negatively related to the real interest rate. If, however, non-separable preferences are assumed, marginal utility may be falling in consumption but need not in output, depending on a suitably large positive u_{CH}. u_{CH} is positive for $\widetilde{\sigma}^{-1} > 1$. In other words, a rise in the real interest rate might induce a change in choices of consumption and leisure the Ricardian agents such that will imply a re-allocation of production in favour of today. Such a scenario would mean an upward-sloping aggregate demand relationship. For $\widetilde{\sigma}^{-1} \neq 1$ and $\lambda = 0$, the inverse slope coefficient Φ^{-1} reduces to

$$\Phi^{-1} = \widetilde{\sigma}^{-1}c^{-1} + \omega\alpha\left(1 - \widetilde{\sigma}^{-1}\right)\omega_H. \tag{D.6}$$

It is easy to show that there is no sufficiently large positive $\widetilde{\sigma}^{-1}$ for which the slope of the aggregate demand relationship $(-\Phi^{-1})$ could become positive. Thus, in an economy without non-Ricardian agents, we do not find an upward-sloping aggregate demand schedule possible. However, when non-Ricardian behaviour is present in the economy, Φ^{-1} becomes

$$\Phi^{-1} = \widetilde{\sigma}^{-1}c^{-1} - \widetilde{\sigma}^{-1}\alpha\Omega_H + \omega\alpha\left(1 - \widetilde{\sigma}^{-1}\right)\chi_H. \tag{D.7}$$

With λ increasing, Ricardian agents enjoy more and more leisure, which amplifies χ_H (or u_{CH}^R), and in the end it comes down to the value of $\widetilde{\sigma}^{-1}$ to determine whether the 'non-separability effect' counteracts or complements the above 'principal' cause of the inverted aggregate demand logic.

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